Bayesian Approaches in Semantics and Pragmatics

An Introduction

Grégoire Winterstein
gregoire.winterstein@linguist.univ-paris-diderot.fr

Aix-Marseille Université, LPL ; HSS, Nanyang Technological University

ERASMUS MUNDUS

April 25, 2014

European Commission
ERASMUS MUNDUS

Pointers, references etc.

- These slides can be found here:
  `http://gregoire.winterstein.free.fr/docs/BayesTutorial.pdf`
- If you have a hard time finding some of the references, contact me.
- Two introductory classes:
  - Notes from a course on probabilistic reasoning and statistical inference for linguists:
    `http://www.stanford.edu/~danlass/NASSLLI-coursenotes-combined.pdf`
  - Course given by S. Dehaene at the *Collège de France* (translated and dubbed in English):
    `http://www.college-de-france.fr/site/en-stanislas-dehaene/course-2012-01-10-09h30.htm`

Pointers, references etc.

- These slides can be found here:
  `http://gregoire.winterstein.free.fr/docs/BayesTutorial.pdf`
- If you have a hard time finding some of the references, contact me.
- Two introductory classes:
  - Notes from a course on probabilistic reasoning and statistical inference for linguists:
    `http://www.stanford.edu/~danlass/NASSLLI-coursenotes-combined.pdf`
  - Course given by S. Dehaene at the *Collège de France* (translated and dubbed in English):
    `http://www.college-de-france.fr/site/en-stanislas-dehaene/course-2012-01-10-09h30.htm`
- **First question**: who knows about Bayes?

The formula that decodes the world!

- Matter
- Climate
- Consciousness
- ...

A revolution for all sciences

The formula that decodes the world!

- Matter
- Climate
- Consciousness
- ...

A revolution for all sciences

- This "revolutionary" formula is Bayes' theorem/law/rule.

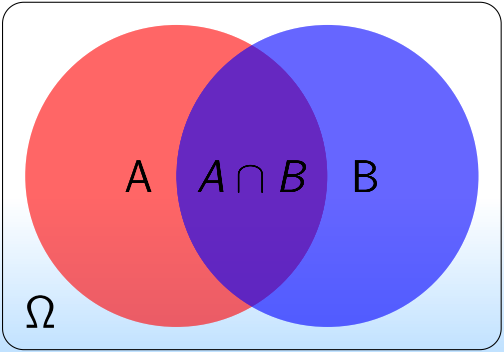
Bayes' Theorem (18th century)

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

Bayes' Theorem (18th century)

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

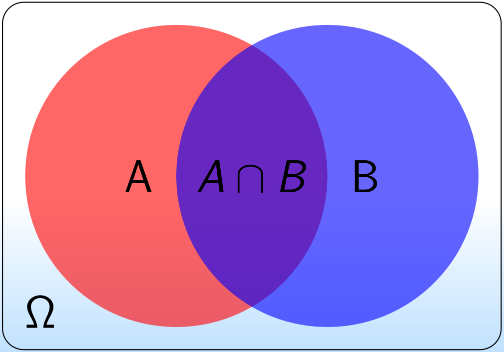
- Very basic result given the axioms of probability theory:
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}$
  - $P(B|A) = \frac{P(A \cap B)}{P(A)}$



Bayes' Theorem (18th century)

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

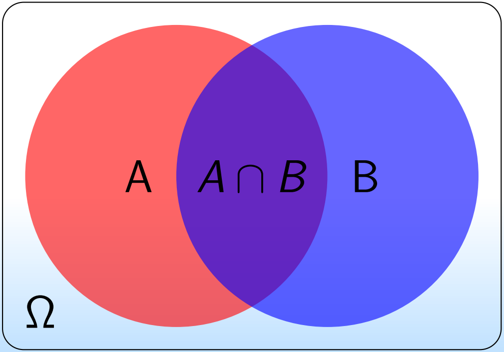
- Very basic result given the axioms of probability theory:
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}$
  - $P(B|A) = \frac{P(A \cap B)}{P(A)}$
- So why all the buzz?



Bayes' Theorem (18th century)

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

- Very basic result given the axioms of probability theory:
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}$
  - $P(B|A) = \frac{P(A \cap B)}{P(A)}$
- So why all the buzz?
- Because of the **Bayesian interpretation of probability** (traceable to Laplace (1812))



Contents

Contents

Contents

Interpreting Probabilities

- There are several ways to interpret the notion of probability:
    1. **Objective** interpretations treat it as the manifestation of a property that is inherent to the studied phenomenon.
    2. The **Bayesian** interpretation treats it in terms of degrees of belief.

Interpreting Probabilities: Objective interpretations

- **Frequency** interpretation: the probability of an event is defined as the relative frequency of the event in some reference class
  - ⇒ No sense in talking of the probability of a non-repeatable event, e.g. *the probability of me dying while giving this tutorial*.

Interpreting Probabilities: Objective interpretations

- **Frequency** interpretation: the probability of an event is defined as the relative frequency of the event in some reference class
  - ⇒ No sense in talking of the probability of a non-repeatable event, e.g. *the probability of me dying while giving this tutorial*.
- **Propensity** interpretation:

  > *Propensities may be explained as possibilities (or as measures of 'weights' of possibilities) which are endowed with tendencies or dispositions to realise themselves, and which are taken to be responsible for the statistical frequencies with which they will in fact realize themselves in long sequences of repetitions of an experiment.* (Popper, 1959)

  - ⇒ Does not shed much light on what probabilities actually are...

Interpreting Probabilities: Objective interpretations

- **Frequency** interpretation: the probability of an event is defined as the relative frequency of the event in some reference class
  - ⇒ No sense in talking of the probability of a non-repeatable event, e.g. *the probability of me dying while giving this tutorial*.
- **Propensity** interpretation:

  > *Propensities may be explained as possibilities (or as measures of 'weights' of possibilities) which are endowed with tendencies or dispositions to realise themselves, and which are taken to be responsible for the statistical frequencies with which they will in fact realize themselves in long sequences of repetitions of an experiment.* (Popper, 1959)

  - ⇒ Does not shed much light on what probabilities actually are...
- Nevertheless, objective interpretations are behind most works and (useful) results in statistics.

Interpreting Probabilities: Bayesian interpretation

- Bayesianism: probability is a measure of **degrees of belief** (Ramsey, 1926).
  - Conditionalization is the key to explain belief update

Interpreting Probabilities: Bayesian interpretation

- Bayesianism: probability is a measure of **degrees of belief** (Ramsey, 1926).
  - Conditionalization is the key to explain belief update
- Several arguments support the Bayesian view:
  - *Dutch book argument*: degrees of belief in complementary events should add up to 1 (for rational agents)
  - Cox (1946) axioms: reasonable axioms about the notion of **plausibility** define a (finitely additive) probability measure (that respects Kolmogorov's axioms).
  - "Linguistic" arguments: cf. later sections.

Interpreting Probabilities: Bayesian interpretation

- Bayesianism: probability is a measure of **degrees of belief** (Ramsey, 1926).
  - Conditionalization is the key to explain belief update
- Several arguments support the Bayesian view:
  - *Dutch book argument*: degrees of belief in complementary events should add up to 1 (for rational agents)
  - Cox (1946) axioms: reasonable axioms about the notion of **plausibility** define a (finitely additive) probability measure (that respects Kolmogorov's axioms).
  - "Linguistic" arguments: cf. later sections.
- **Issues**: how does one learn probabilities? What are the "exact numbers"?

Interpreting Bayes' rule

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$$

H: a Hypothesis

E: a piece of Evidence

- Bayes' rule is a way of evaluating how much a new observation affects our degree of belief in a given hypothesis.

Interpreting Bayes' rule

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$$

H: a Hypothesis

E: a piece of Evidence

- Bayes' rule is a way of evaluating how much a new observation affects our degree of belief in a given hypothesis.
  - $P(H)$ is the **prior** belief in H

Interpreting Bayes' rule

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$$

H: a Hypothesis

E: a piece of Evidence

- Bayes' rule is a way of evaluating how much a new observation affects our degree of belief in a given hypothesis.
  - $P(H)$ is the **prior** belief in H
  - $P(E|H)$ is the **likelihood** of observing the effect E, assuming that H is true

Interpreting Bayes' rule

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$$

H: a Hypothesis

E: a piece of Evidence

- Bayes' rule is a way of evaluating how much a new observation affects our degree of belief in a given hypothesis.
  - $P(H)$ is the **prior** belief in H
  - $P(E|H)$ is the **likelihood** of observing the effect E, assuming that H is true
  - $P(E)$ is seldom discussed as such (or at all), and is usually rewritten as: $P(E|H) \times P(H) + P(E|\neg H) \times P(\neg H)$

A qualitative example

- You see Bob coughing:
  - H_1: Bob has the flu
  - H_2: Bob has lung cancer
  - H_3: Bob has gastroenteritis

A qualitative example

- You see Bob coughing:
  - H_1: Bob has the flu
  - H_2: Bob has lung cancer
  - H_3: Bob has gastroenteritis
- $P(H_i|\text{Cough}) \propto P(\text{Cough}|H_i) \times P(H_i)$

	Likelihood	*Prior*
H_1	High	High
H_2	High	Low
H_3	Low	High

A qualitative example

- You see Bob coughing:
  - H_1: Bob has the flu
  - H_2: Bob has lung cancer
  - H_3: Bob has gastroenteritis
- $P(H_i|\text{Cough}) \propto P(\text{Cough}|H_i) \times P(H_i)$

	Likelihood	*Prior*
H_1	High	High
H_2	High	Low
H_3	Low	High

- H_1 is the most probable hypothesis **a posteriori**.

A qualitative example

- You see Bob coughing:
  - H_1: Bob has the flu
  - H_2: Bob has lung cancer
  - H_3: Bob has gastroenteritis
- $P(H_i|\text{Cough}) \propto P(\text{Cough}|H_i) \times P(H_i)$

	Likelihood	*Prior*
H_1	High	High
H_2	High	Low
H_3	Low	High

- H_1 is the most probable hypothesis **a posteriori**.
- A maximum likelihood based approach might have selected H_2
- The choice of H_1 is the result of an **abductive** reasoning: H_1 is the hypothesis that best explains the observation (i.e. the coughing).

An exercise (Gigerenzer, 1991)

- A certain disease affects about 1 person in 1000.
- Doctors devised a test for the disease:
    - On average, out of 100 people that have the disease, 99 will get a positive result.
    - On average, out of 100 people that do not have the disease, 99 will get a negative result.
- Is the test a good one? Should you be worried if you test positive?

An exercise (Gigerenzer, 1991)

- A certain disease affects about 1 person in 1000.
- Doctors devised a test for the disease:
  - On average, out of 100 people that have the disease, 99 will get a positive result.
  - On average, out of 100 people that do not have the disease, 99 will get a negative result.
- Is the test a good one? Should you be worried if you test positive?

- Let $H =$*I have the disease* and $E =$*the test is positive*.
- We need to calculate $P(H|E) = \frac{P(E|H) \times P(H)}{P(E)}$
- $P(E|H) = 0.99$, $P(H) = 0.001$
- $P(E) = P(E|H) \times P(H) + P(E|\neg H) \times P(\neg H) = 0.99 \times 1/1000 + 1/100 \times 999/1000 = 0.1098$
- Therefore $P(H|E) \approx 0.09$ ...

Why a tutorial on Bayes in linguistics?

- Because Bayesian insights can be applied to problems in linguistics in various domains:
  - acquisition
  - phonology
  - morphology
  - syntax
  - semantics
  - pragmatics
  - language production and interpretation
  - ...

Why a tutorial on Bayes in linguistics?

- Because Bayesian insights can be applied to problems in linguistics in various domains:
  - acquisition
  - phonology
  - morphology
  - syntax
  - semantics
  - pragmatics
  - language production and interpretation
  - ...
- Because Bayesian approaches are effective to model **human reasoning** and **human perception** (Oaksford & Chater, 2007; Tenenbaum et al., 2011).
  - Strong ties with semantics, pragmatics and knowledge representation.

Contents

Bayesian inference

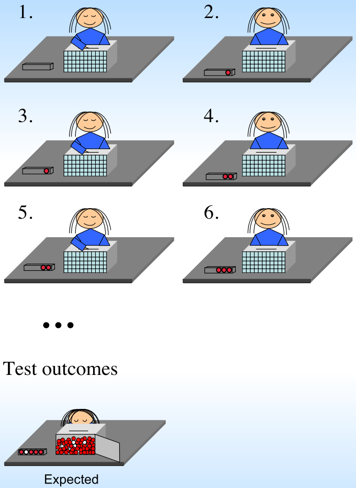
- Bayesian inferences are made at every cognitive level

Bayesian inference

- Bayesian inferences are made at every cognitive level
- Xu & Garcia (2008): **8 months old** infants can make such inferences

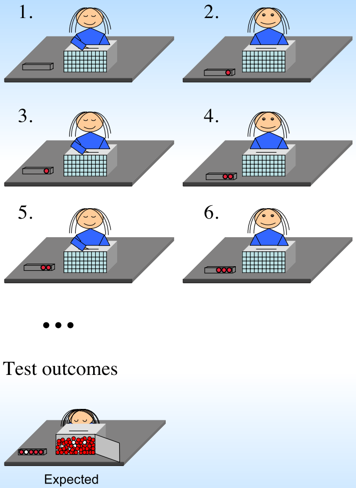
Bayesian inference

- Bayesian inferences are made at every cognitive level
- Xu & Garcia (2008): **8 months old** infants can make such inferences



Bayesian inference

- Bayesian inferences are made at every cognitive level
- Xu & Garcia (2008): **8 months old** infants can make such inferences
- The child estimated the distribution of the balls in the urn based only on a few observations:
  $P(H|E) \propto P(E|H) \times P(H)$
  - H: distribution of the balls
  - E: observations



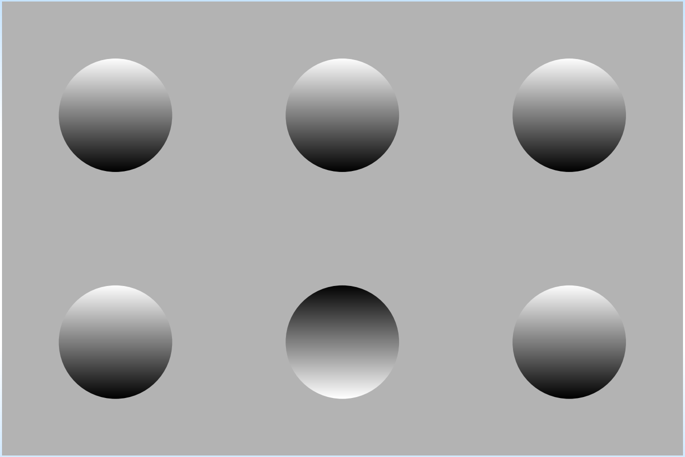
Bayesian inference in vision

http://www.york.ac.uk/depts/maths/histstat/bayespic.htm

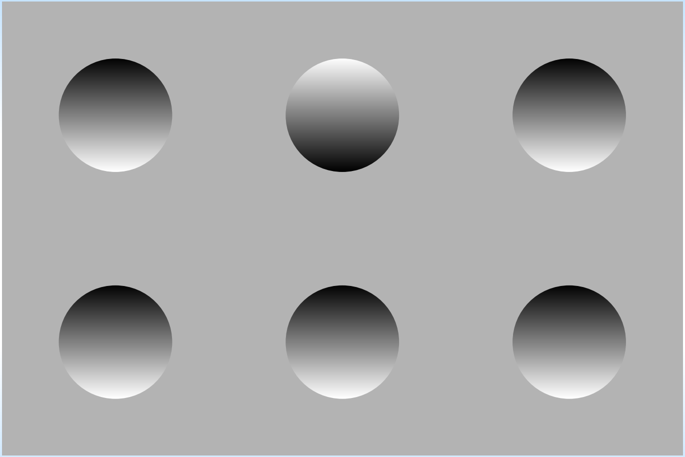
Bayesian inference in vision

http://www.york.ac.uk/depts/maths/histstat/bayespic.htm

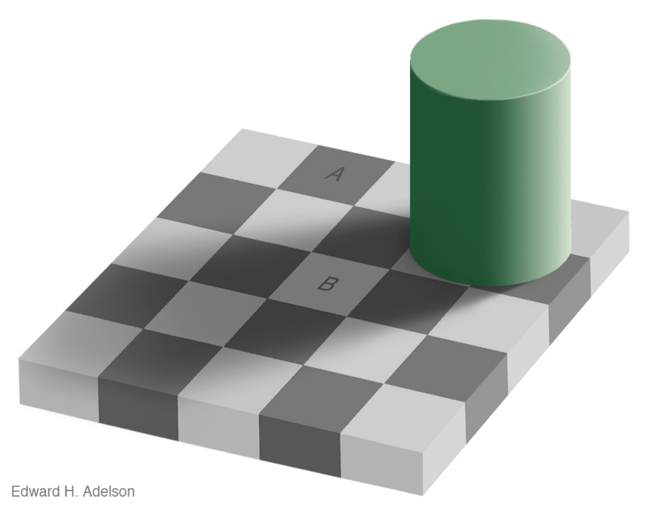
Bayesian inference in vision



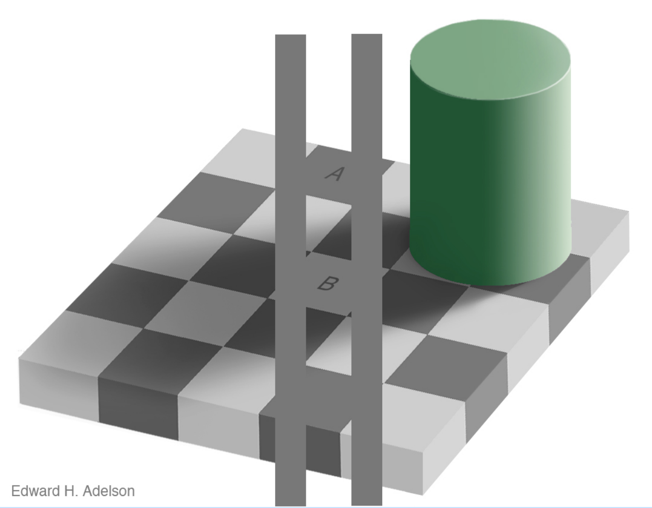
Bayesian inference in vision



Bayesian Inference in Vision (II)

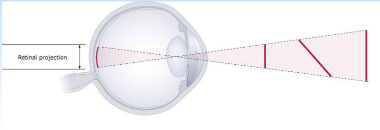


Bayesian Inference in Vision (II)



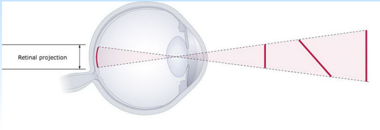
Explaining Vision

- Sensorial input is almost always ambiguous



Explaining Vision

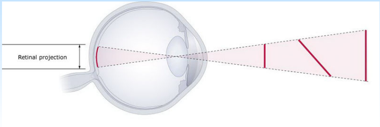
- Sensorial input is almost always ambiguous



- We select the most plausible interpretation based on:
  1. The **prior** knowledge about objects in the world (accumulation of knowledge through learning), e.g.:
     - Most probable source of light
     - Effects of shadowing and borders in tiles

Explaining Vision

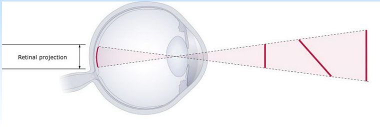
- Sensorial input is almost always ambiguous



- We select the most plausible interpretation based on:
  1. The **prior** knowledge about objects in the world (accumulation of knowledge through learning), e.g.:
     - Most probable source of light
     - Effects of shadowing and borders in tiles
  2. The knowledge of the **likelihood** of an observation given an hypothesis (based on an internal model of "how things work"):
     - Likelihood of being concave/convex knowing the light comes from above and given a light gradient
     - Likelihood of being different shades of grey knowing one is in the shadow and not the other

Explaining Vision

- Sensorial input is almost always ambiguous



- We select the most plausible interpretation based on:
  1. The **prior** knowledge about objects in the world (accumulation of knowledge through learning), e.g.:
     - Most probable source of light
     - Effects of shadowing and borders in tiles
  2. The knowledge of the **likelihood** of an observation given an hypothesis (based on an internal model of “how things work”):
     - Likelihood of being concave/convex knowing the light comes from above and given a light gradient
     - Likelihood of being different shades of grey knowing one is in the shadow and not the other
  3. **Bayes’ rule** which selects the most likely interpretation based on priors and likelihoods

Induction scandal

- More generally, a Bayesian approach gives an answer to the (old) **problem of induction**:

> *For scientists studying how humans come to understand their world, the central challenge is this: How do our minds get so much from so little?* (Tenenbaum et al., 2011)

Induction scandal

- More generally, a Bayesian approach gives an answer to the (old) **problem of induction**:

> *For scientists studying how humans come to understand their world, the central challenge is this: How do our minds get so much from so little?* *(Tenenbaum et al., 2011)*

- Examples:
  - The examples presented before
  - Causal relations based on few observations
  - The **gavagai** story
  - Chomsky's "argument" about the poverty of stimulus
  - . . .

Fast induction (Schmidt, 2009)

- Objects in red are **tufa**, where are the other tufas?

Fast induction (Schmidt, 2009)

- Objects in red are **tufa**, where are the other tufas?

Fast induction (cont.)

- Hypotheses are about which categories words could label and they correspond to branches of the tree
- Priors are proportional to the height of the branch
- Likelihood favors the most specific categories by assuming that examples are drawn randomly from the branch the word labels

Bayesian Networks

- Probabilistic knowledge is often represented in the form of Bayesian networks
- **Advantages** (Oaksford & Chater, 2007, pp. 84–88):
  - Such models directly represent knowledge in a compact way, unlike other approaches in AI that focus on the reasoning processes rather than on the knowledge itself.
  - Typical models are “local” and the connections are sparse: they are tractable and can be manually updated with expert knowledge.
  - These models can be automatically learned from data, both in terms of structure and strength of the links

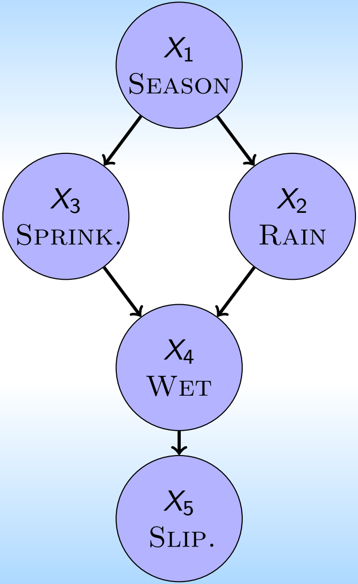
Bayesian Networks

- Probabilistic knowledge is often represented in the form of Bayesian networks
- **Advantages** (Oaksford & Chater, 2007, pp. 84–88):
  - Such models directly represent knowledge in a compact way, unlike other approaches in AI that focus on the reasoning processes rather than on the knowledge itself.
  - Typical models are "local" and the connections are sparse: they are tractable and can be manually updated with expert knowledge.
  - These models can be automatically learned from data, both in terms of structure and strength of the links
- These networks also represent a way to make explicit causality assumptions (Pearl, 2009).

Bayesian Networks

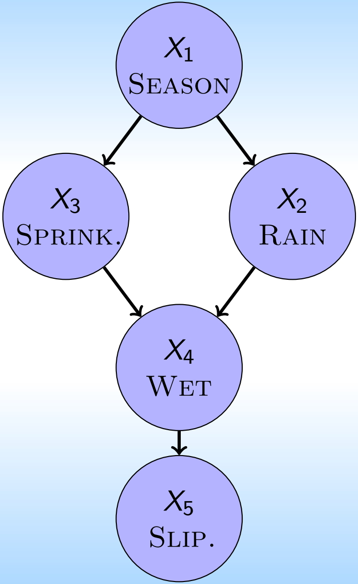
- Probabilistic knowledge is often represented in the form of Bayesian networks
- **Advantages** (Oaksford & Chater, 2007, pp. 84–88):
  - Such models directly represent knowledge in a compact way, unlike other approaches in AI that focus on the reasoning processes rather than on the knowledge itself.
  - Typical models are "local" and the connections are sparse: they are tractable and can be manually updated with expert knowledge.
  - These models can be automatically learned from data, both in terms of structure and strength of the links
- These networks also represent a way to make explicit causality assumptions (Pearl, 2009).
- Recent approaches use more sophisticated representations such as Hierarchical Bayesian Models (Tenenbaum et al., 2011).

Network example



- Season (X_1) affects whether it rains (X_2) or whether sprinklers are on (X_3)
- The wetness of the pavement (X_4) is affected by rain and the status of the sprinklers
- The wetness of the pavement affects its slipperiness (X_5)

Network example



- Season (X_1) affects whether it rains (X_2) or whether sprinklers are on (X_3)
- The wetness of the pavement (X_4) is affected by rain and the status of the sprinklers
- The wetness of the pavement affects its slipperiness (X_5)
- Causality assumptions are in the **absence** of arrows:
  - Rain does not affect the season
  - ...
- There are criteria to test causality assumptions (Pearl, 2009)

The Bayesian approach in Semantics and Pragmatics

- Several trends in the broad Bayesian picture in semantics and pragmatics:
    1. A Bayesian approach to **reasoning** that can be applied to some specific constructions such as **conditionals**
       (Oaksford & Chater, 2007, 2010)

The Bayesian approach in Semantics and Pragmatics

- Several trends in the broad Bayesian picture in semantics and pragmatics:
  1. A Bayesian approach to **reasoning** that can be applied to some specific constructions such as **conditionals** (Oaksford & Chater, 2007, 2010)
  2. A "weakly" Bayesian approach that considers **meaning** to be **probabilistic** and deals with degrees of belief (Lassiter, 2011b; Goodman & Lassiter, 2014)

The Bayesian approach in Semantics and Pragmatics

- Several trends in the broad Bayesian picture in semantics and pragmatics:
    1. A Bayesian approach to **reasoning** that can be applied to some specific constructions such as **conditionals** (Oaksford & Chater, 2007, 2010)
    2. A "weakly" Bayesian approach that considers **meaning** to be **probabilistic** and deals with degrees of belief (Lassiter, 2011b; Goodman & Lassiter, 2014)
    3. A "strongly" Bayesian approach to **natural language interpretation** that makes a central use of Bayes' formula and its interpretation (Winterstein, 2012; Zeevat, 2014)

Contents

The Selection Task (Wason, 1966)

- Four double-sided cards with:
  - A number on one side
  - A colour on the other

The Selection Task (Wason, 1966)

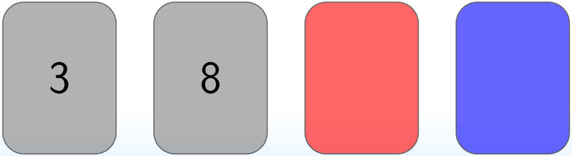
- Four double-sided cards with:
  - A number on one side
  - A colour on the other
- Which cards to turn over in order to test the following hypothesis?

If a card has an 8 on one side, the other side is red.

The Selection Task (Wason, 1966)

- Four double-sided cards with:
  - A number on one side
  - A colour on the other
- Which cards to turn over in order to test the following hypothesis?

If a card has an 8 on one side, the other side is red.



The semantics of conditionals

- The classical semantic analysis of a conditional statement uses material conditionals from propositional logic.
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

The semantics of conditionals

- The classical semantic analysis of a conditional statement uses material conditionals from propositional logic.
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	**0**	**0**
1	1	1

- To test a rule, one should (logically) try to falsify it

The semantics of conditionals

- The classical semantic analysis of a conditional statement uses material conditionals from propositional logic.
- Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	**0**	**0**
1	1	1

- To test a rule, one should (logically) try to falsify it
- The logical choice of cards would be the ones that instantiate the only case where the conditional is false:

  $\Rightarrow$ The `8` / `non red` cases

Conditionals

- Subjects draw **logically invalid** inferences from conditionals.
- Furthermore, the inferences drawn depend on the task. **Modus Tollens** is endorsed by a majority subjects when asked about its validity but:
  - Less frequently than **Modus Ponens**
  - Invalid inferences (*Denying the Antecedent* and *Affirming the Consequent*) are also endorsed by a majority of subjects
  - Subjects seem unable to perform MT in some cases (cf. Wason task).
- Several theories try to account for the data by considering it as evidence for the **limitations** of the cognitive system:
  - Mental Logic (Rips, 1994)
  - Mental Models (Johnson-Laird, 1983; Johnson-Laird & Byrne, 2002)
- But none fits all the existing data (Oaksford & Chater, 2007).

Bayesian conditional reasoning (Oaksford & Chater, 2003, 2007, 2010)

- The Bayesian consider the results of the Wason task not as "errors", but as normal **features** of the reasoning system.

Bayesian conditional reasoning (Oaksford & Chater, 2003, 2007, 2010)

- The Bayesian consider the results of the Wason task not as "errors", but as normal **features** of the reasoning system.
- Basic tenets of the Bayesian approach to conditional reasoning:
  1. The probability of conditionals is **conditional probability**: $P(\text{if p then q}) = P(q|p)$
  2. Probabilities are degrees of belief
  3. Conditional probabilities are determined by the Ramsay test: make the hypothesis that p and adjust your belief in q accordingly

Bayesian conditional reasoning (Oaksford & Chater, 2003, 2007, 2010)

- The Bayesian consider the results of the Wason task not as "errors", but as normal **features** of the reasoning system.
- Basic tenets of the Bayesian approach to conditional reasoning:
  1. The probability of conditionals is **conditional probability**: $P(\text{if p then q}) = P(q|p)$
  2. Probabilities are degrees of belief
  3. Conditional probabilities are determined by the Ramsay test: make the hypothesis that p and adjust your belief in q accordingly
  4. **Conditionalization**: upon learning that p the belief in q should be equal to $P(q|p)$.

Bayesian conditional reasoning (Oaksford & Chater, 2003, 2007, 2010)

- The Bayesian consider the results of the Wason task not as "errors", but as normal **features** of the reasoning system.
- Basic tenets of the Bayesian approach to conditional reasoning:
  1. The probability of conditionals is **conditional probability**: $P(\text{if p then q}) = P(q|p)$
  2. Probabilities are degrees of belief
  3. Conditional probabilities are determined by the Ramsay test: make the hypothesis that p and adjust your belief in q accordingly
  4. **Conditionalization**: upon learning that p the belief in q should be equal to $P(q|p)$.
- **Example**:
  - You believe that $P($*If it is sunny in Wimbledon, then John plays tennis*$) = 0.9$
  - You learn it is sunny in Wimbledon
  - Then your belief in *John plays tennis* is 0.9.

Explaining the Wason Task in Bayesian terms

- The goal of the Wason task is to test a conditional hypothesis H.
- The hypothesis is interpreted as claiming that the **probability of the conditional is high**.
- **Example:** *If there is an 8 on one side, the other side is red* means that the probability of a card being red knowing it is an 8 is higher than just that of it being red.

Explaining the Wason Task in Bayesian terms

- The goal of the Wason task is to test a conditional hypothesis H.
- The hypothesis is interpreted as claiming that the **probability of the conditional is high**.
- **Example:** *If there is an 8 on one side, the other side is red* means that the probability of a card being red knowing it is an 8 is higher than just that of it being red.
- It is assumed subjects **compare hypotheses** rather than try to falsify the one presented to them, i.e. they compare:
  - H_D: $P(q|p) > P(p)$
  - H_0: $P(q|p) = P(p)$

Explaining the Wason Task in Bayesian terms

- The goal of the Wason task is to test a conditional hypothesis H.
- The hypothesis is interpreted as claiming that the **probability of the conditional is high**.
- **Example:** *If there is an 8 on one side, the other side is red* means that the probability of a card being red knowing it is an 8 is higher than just that of it being red.
- It is assumed subjects **compare hypotheses** rather than try to falsify the one presented to them, i.e. they compare:
  - H_D: $P(q|p) > P(p)$
  - H_0: $P(q|p) = P(p)$
- Initially, it is assumed subjects are **neutral** regarding the rules: $P(H_D) = P(H_0) = 0.5$.

Explaining Wason (cont.)

- To tease apart H_D and H_0 subjects choose the cards that yield the best **informative gain** (Oaksford & Chater, 2003).

Explaining Wason (cont.)

- To tease apart H_D and H_0 subjects choose the cards that yield the best **informative gain** (Oaksford & Chater, 2003).
- A card is informative if it reduces the uncertainty between H_D and H_0
  - Uncertainty is measured via Shannon-Wiener information
    - Maximum uncertainty equals 1 bit: H_D and H_0 are equally likely
    - Minimal uncertainty equals 0 bit: either H_D or H_0 has probability of 1

Explaining Wason (cont.)

- To tease apart H_D and H_0 subjects choose the cards that yield the best **informative gain** (Oaksford & Chater, 2003).
- A card is informative if it reduces the uncertainty between H_D and H_0
  - Uncertainty is measured via Shannon-Wiener information
    - Maximum uncertainty equals 1 bit: H_D and H_0 are equally likely
    - Minimal uncertainty equals 0 bit: either H_D or H_0 has probability of 1
- Suppose the 8 card $(= p)$ is chosen and that the other side is red $(= q)$.
  - **Bayes' rule**: $P(H_i|p \wedge q) = \frac{P(p \wedge q|H_i)}{P(p \wedge q)}$
  - This can be calculated as a function of $P(p)$, $P(q)$ and $P(q|p)$, and the prior probabilities of H_i
  - $P(q|p)$ is the quantity that differentiates H_0 and H_D

Explaining Wason (cont.)

- To tease apart H_D and H_0 subjects choose the cards that yield the best **informative gain** (Oaksford & Chater, 2003).
- A card is informative if it reduces the uncertainty between H_D and H_0
  - Uncertainty is measured via Shannon-Wiener information
    - Maximum uncertainty equals 1 bit: H_D and H_0 are equally likely
    - Minimal uncertainty equals 0 bit: either H_D or H_0 has probability of 1
- Suppose the 8 card ($= p$) is chosen and that the other side is red ($= q$).
  - **Bayes' rule**: $P(H_i|p \wedge q) = \frac{P(p \wedge q|H_i)}{P(p \wedge q)}$
  - This can be calculated as a function of $P(p)$, $P(q)$ and $P(q|p)$, and the prior probabilities of H_i
  - $P(q|p)$ is the quantity that differentiates H_0 and H_D
- By considering the possible outcomes for each card, one can calculate their **expected information gains**.

Calculating informativity: the details

	q	$\neg q$	*Marginal*
p	$a(1-\varepsilon)$	$a\varepsilon$	a
$\neg p$	$b-a(1-\varepsilon)$	$(1-b)-a\varepsilon$	$1-a$
Marginal	b	$(1-b)$	

Table: Contingencies under H_D (for H_0 cell values are the product of marginal probabilities) $a = P(p), b = P(q), \varepsilon = P(\neg q|p, H_D)$

Calculating informativity: the details

	q	$\neg q$	*Marginal*
p	$a(1-\varepsilon)$	$a\varepsilon$	a
$\neg p$	$b-a(1-\varepsilon)$	$(1-b)-a\varepsilon$	$1-a$
Marginal	b	$(1-b)$	

Table: Contingencies under H_D (for H_0 cell values are the product of marginal probabilities) $a = P(p), b = P(q), \varepsilon = P(\neg q|p, H_D)$

- Suppose we find q on the other side of p:
- $P(H_D|p \wedge q) = \frac{P(p \wedge q|H_D) \times P(H_D)}{P(p \wedge q)}$
- $= \frac{a(1-\varepsilon) \times 0.5}{P(p \wedge q|H_D)P(H_D) + P(p \wedge q|H_0)P(H_0)}$
- $= \frac{a(1-\varepsilon) \times 0.5}{a(1-\varepsilon) \times 0.5 + ab \times 0.5}$

Calculating informativity: the details

	q	$\neg q$	*Marginal*
p	$a(1-\varepsilon)$	$a\varepsilon$	a
$\neg p$	$b - a(1-\varepsilon)$	$(1-b) - a\varepsilon$	$1-a$
Marginal	b	$(1-b)$	

Table: Contingencies under H_D (for H_0 cell values are the product of marginal probabilities) $a = P(p), b = P(q), \varepsilon = P(\neg q|p, H_D)$

- Suppose we find q on the other side of p:
- $P(H_D|p \wedge q) = \frac{P(p \wedge q|H_D) \times P(H_D)}{P(p \wedge q)}$
- $= \frac{a(1-\varepsilon) \times 0.5}{P(p \wedge q|H_D)P(H_D) + P(p \wedge q|H_0)P(H_0)}$
- $= \frac{a(1-\varepsilon) \times 0.5}{a(1-\varepsilon) \times 0.5 + ab \times 0.5}$

- If $a = 0.2,\ b = 0.3,\ \varepsilon = 0.9$
- $P(H_D|p \wedge q) = 0.75$ and $P(H_0|p \wedge q) = 0.25$
- New entropy value: 0.81 bits, information gain: 0.19 bits.

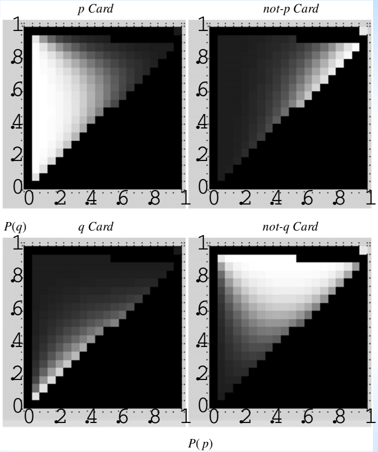
Calculating informativity: the details (II)

- The expected information gain of a card (e.g. the `8` card) depends on both possibilities for the other side (e.g. `red` / `not red`).
- The expected information is averaged over both possibilities, weighted by the prior probabilities of each outcome:

  $EI(p) = P(q|p) \times IG(p \wedge q) + P(\neg q|p) \times IG(p \wedge \neg q)$

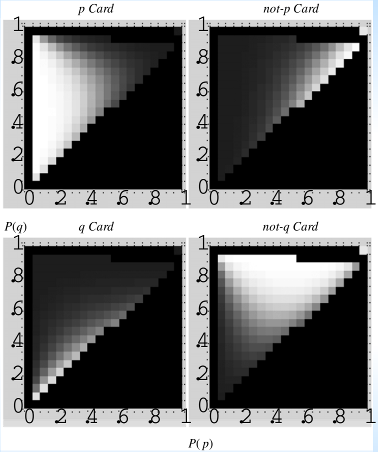
  - Where: $P(q|p) = P(H_D)P(q|p \wedge H_D) + P(H_0)P(q|p \wedge H_0) = P(H_D)P(q|p \wedge H_D) + P(H_0)P(q|H_0)$
- The expected information gain $EI_g(p)$ is the difference between the initial information and $EI(p)$, and is further scaled by the total amount of information available in the setup.
  - $SEI_g(x) = \frac{EI_g(x)}{\sum_{x_i \in \{p, \neg p, q, \neg q\}} EI_g(x_i)}$

Output: Expected Information Gains



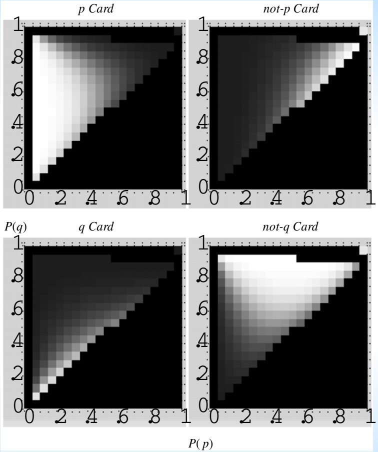
- The model estimates the **Expected Information Gain** for each card as a function of $P(p)$ and $P(q)$
- The left-hand figures show that each card is informative for some combinations of priors

Output: Expected Information Gains



- The model estimates the **Expected Information Gain** for each card as a function of $P(p)$ and $P(q)$
- The left-hand figures show that each card is informative for some combinations of priors
- Why the preference for the q card over the $\neg q$?

Output: Expected Information Gains



- The model estimates the **Expected Information Gain** for each card as a function of $P(p)$ and $P(q)$
- The left-hand figures show that each card is informative for some combinations of priors
- Why the preference for the q card over the $\neg q$?
- O&C's answer: **rarity assumption**
  - Categories of natural language divide the world up finely

  $\Rightarrow$ $P(p)$ and $P(q)$ are intuitively "low" (Anderson & Sheu, 1995; McKenzie & Mikkelsen, 2000)

Rarity assumption: an illustration

(1) If a person is bitten by a vampire bat, they will develop pointed teeth.

- Who do you check to see whether (1) is true?

Rarity assumption: an illustration

(1) If a person is bitten by a vampire bat, they will develop pointed teeth.

- Who do you check to see whether (1) is true?
- People who have been bitten, to see if they have pointed teeth

Rarity assumption: an illustration

(1) If a person is bitten by a vampire bat, they will develop pointed teeth.

- Who do you check to see whether (1) is true?
- People who have been bitten, to see if they have pointed teeth
- People who have pointed teeth: learning that they have been bitten will improve the belief in (1)

Rarity assumption: an illustration

(1) If a person is bitten by a vampire bat, they will develop pointed teeth.

- Who do you check to see whether (1) is true?
- People who have been bitten, to see if they have pointed teeth
- People who have pointed teeth: learning that they have been bitten will improve the belief in (1)
- Checking people without pointed teeth is not productive:
  - Most will not have been bitten
  - Therefore, the expected information gain is very small

Selection Task: alternative version

- Another deck of cards, representing people at a party:
  - An age on one side
  - A drink on the other
- Hypothesis to test:

  If somebody drinks alcohol, he must be at least 18.

- Which cards should be turned over?

Selection Task: alternative version

- Another deck of cards, representing people at a party:
  - An age on one side
  - A drink on the other
- Hypothesis to test:

  If somebody drinks alcohol, he must be at least 18.

- Which cards should be turned over?

Gin	Kopi	35	15

Wason: the deontic case

- People appear much more “logical” when the task deals with obligations, i.e. with **deontic** rules.
- There is little point in confirming/denying a rule: the rule just is.
- Therefore subjects attempt to find whether the rule is **disobeyed**
- This entails different strategies, close to “Popperian falsification”.
- It also partly undermines the competitive appraoches’ claims that we are unable to realize *Modus Tollens* inferences because of limitations of our cognitive system.
- More details: Oaksford & Chater (2007).

Taking stock

- The Bayesian approach to reasoning:
  - **Rejects the logical analysis** of conditional sentences as material conditionals.
  - Postulates that the probability of a conditional is **conditional probability**
  - Correctly predicts that some subjects endorse **(logically) invalid inference patterns**.
  - Correctly predicts that the results of the Wason task:
    - Depend on the **prior probabilities** of antecedent and consequent
    - Depend on the **modal flavor** of the rule to be tested

Taking stock

- The Bayesian approach to reasoning:
  - **Rejects the logical analysis** of conditional sentences as material conditionals.
  - Postulates that the probability of a conditional is **conditional probability**
  - Correctly predicts that some subjects endorse **(logically) invalid inference patterns**.
  - Correctly predicts that the results of the Wason task:
    - Depend on the **prior probabilities** of antecedent and consequent
    - Depend on the **modal flavor** of the rule to be tested
- This approach can be applied to phenomena beyond conditionals:
  - Syllogisms
  - Argumentation (cf. infra)
  - . . .

Contents

Contents

Contents

The switch to a probabilistic model

- Recent works argued for a probabilistic treatment of meaning:

The switch to a probabilistic model

- Recent works argued for a probabilistic treatment of meaning:
  - Results from the literature on **cognition** strongly suggest our cognitive system deals with degrees of belief (cf. previous section)

The switch to a probabilistic model

- Recent works argued for a probabilistic treatment of meaning:
  - Results from the literature on **cognition** strongly suggest our cognitive system deals with degrees of belief (cf. previous section)
  - Carnap (1950), and more recently Merin (1999), argue that a probabilistic approach to meaning is a better approximation of **human beliefs** than one based solely on truth-conditions.

The switch to a probabilistic model

- Recent works argued for a probabilistic treatment of meaning:
  - Results from the literature on **cognition** strongly suggest our cognitive system deals with degrees of belief (cf. previous section)
  - Carnap (1950), and more recently Merin (1999), argue that a probabilistic approach to meaning is a better approximation of **human beliefs** than one based solely on truth-conditions.
  - Yalcin (2007); Lassiter (2011b) argue for a probabilistic treatment of meaning based on the case of **gradable epistemic modals**.

The switch to a probabilistic model

- Recent works argued for a probabilistic treatment of meaning:
  - Results from the literature on **cognition** strongly suggest our cognitive system deals with degrees of belief (cf. previous section)
  - Carnap (1950), and more recently Merin (1999), argue that a probabilistic approach to meaning is a better approximation of **human beliefs** than one based solely on truth-conditions.
  - Yalcin (2007); Lassiter (2011b) argue for a probabilistic treatment of meaning based on the case of **gradable epistemic modals**.
- These approaches are Bayesian in as much as they equate probabilities with degrees of belief. Bayes' rule itself is not necessarily central in those accounts.

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations. . .)

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations...)
  - Worlds are related by a compatibility relation.

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations...)
  - Worlds are related by a compatibility relation.
  - A proposition is a set of worlds: the worlds in which the proposition is true.

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations...)
  - Worlds are related by a compatibility relation.
  - A proposition is a set of worlds: the worlds in which the proposition is true.
- A **probability measure** is added to the basic ontology: it represents the speaker's degrees of belief:

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations. . .)
  - Worlds are related by a compatibility relation.
  - A proposition is a set of worlds: the worlds in which the proposition is true.
- A **probability measure** is added to the basic ontology: it represents the speaker's degrees of belief:
  - The sum of the probabilities of individual worlds is 1.

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations...)
  - Worlds are related by a compatibility relation.
  - A proposition is a set of worlds: the worlds in which the proposition is true.
- A **probability measure** is added to the basic ontology: it represents the speaker's degrees of belief:
  - The sum of the probabilities of individual worlds is 1.
  - The probability of a proposition is the probability of the corresponding set of worlds.

Probabilistic meaning in a nutshell

- Most models are based on **intensional logic**:
  - Basic ontology: set of information points (worlds, situations...)
  - Worlds are related by a compatibility relation.
  - A proposition is a set of worlds: the worlds in which the proposition is true.
- A **probability measure** is added to the basic ontology: it represents the speaker's degrees of belief:
  - The sum of the probabilities of individual worlds is 1.
  - The probability of a proposition is the probability of the corresponding set of worlds.
- **Belief update** is modeled by conditioning: upon learning that φ is true, the probability measure P is replaced by P' such that $\lambda x.P'(x) = \lambda x.P(x|\varphi)$.

Gradable Epistemic Modals

- The standard theory of modality (Kratzer, 1991) licenses the following pattern of inference for an epistemic modal like *likely*:

(2) a. φ is as likely as ψ.
  b. φ is as likely as χ.
  c. $\therefore \varphi$ is as likely as $(\psi \vee \chi)$.

Gradable Epistemic Modals

- The standard theory of modality (Kratzer, 1991) licenses the following pattern of inference for an epistemic modal like *likely*:

(2) a. φ is as likely as ψ.
b. φ is as likely as χ.
c. $\therefore \varphi$ is as likely as $(\psi \vee \chi)$.

- This pattern is wrong:

(3) There is a lottery with 1000 tickets. People can buy only one ticket. Lemmy, Ritchie (and many others) bought tickets.
a. $\forall x$: Lemmy is as likely to win as x.
b.
c.
d.
e.

Gradable Epistemic Models

- The standard theory of modality (Kratzer, 1991) licenses the following pattern of inference for an epistemic modal like *likely*:

(2) a. φ is as likely as ψ.
b. φ is as likely as χ.
c. $\therefore \varphi$ is as likely as $(\psi \vee \chi)$.

- This pattern is wrong:

(3) There is a lottery with 1000 tickets. People can buy only one ticket. Lemmy, Ritchie (and many others) bought tickets.
a. $\forall x$: Lemmy is as likely to win as x.
b. Let $q_i = x_i$ wins the lottery, $p =$*Lemmy wins the lottery* and $\geq =$*is as likely to win*
c. $(p \geq q_0) \wedge (p \geq q_1) \ldots (p \geq q_{998})$
d.
e.

Gradable Epistemic Modals

- The standard theory of modality (Kratzer, 1991) licenses the following pattern of inference for an epistemic modal like *likely*:

(2)
a. φ is as likely as ψ.
b. φ is as likely as χ.
c. $\therefore \varphi$ is as likely as $(\psi \vee \chi)$.

- This pattern is wrong:

(3) There is a lottery with 1000 tickets. People can buy only one ticket. Lemmy, Ritchie (and many others) bought tickets.
a. $\forall x$: Lemmy is as likely to win as x.
b. Let $q_i = x_i$ wins the lottery, $p =$*Lemmy wins the lottery* and $\geq$=*is as likely to win*
c. $(p \geq q_0) \wedge (p \geq q_1) \ldots (p \geq q_{998})$
d. Apply (2-c): $p \geq (q_0 \vee q_1 \cdots \vee q_{998})$
e. $\Rightarrow$ Lemmy is as likely to win as he is not to win.

Gradable Epistemic Modals (II)

- Epistemic adjectives such as *likely* and *probable* are **gradable** (Kennedy & McNally, 2005):

(4) a. It is very likely that Lemmy plays the bass.
b. It more probable that Lemmy plays the bass than the piano.

Gradable Epistemic Modals (II)

- Epistemic adjectives such as *likely* and *probable* are **gradable** (Kennedy & McNally, 2005):

(4) a. It is very likely that Lemmy plays the bass.
b. It more probable that Lemmy plays the bass than the piano.

- Lassiter (2011b,a) show that these adjectives are associated with an additive measure equivalent to a probability measure (which crucially, **does not validate (2-c)**):

(5) a. φ is *possible* iff $P(\varphi) > 0$
b. φ is more likely that ψ iff $P(\varphi) > P(\psi)$

Gradable Epistemic Modals (II)

- Epistemic adjectives such as *likely* and *probable* are **gradable** (Kennedy & McNally, 2005):

(4) a. It is very likely that Lemmy plays the bass.
    b. It more probable that Lemmy plays the bass than the piano.

- Lassiter (2011b,a) show that these adjectives are associated with an additive measure equivalent to a probability measure (which crucially, **does not validate (2-c)**):

(5) a. φ is *possible* iff $P(\varphi) > 0$
    b. φ is more likely that ψ iff $P(\varphi) > P(\psi)$

- Lassiter (2012b) claims that this suggests that the mathematics of **probability is discernible in language**, i.e. that "*a knowledge of probability must form part of our knowledge of the semantics of the English language*".

Contents

Presupposition

- **Presupposition** is part of the total meaning conveyed by an utterance.
- There is a huge literature on the matter: (Frege, 1892; Russell, 1905; Strawson, 1950; Ducrot, 1972; Stalnaker, 1974; Karttunen, 1974; Karttunen & Peters, 1979; Lewis, 1979; Gazdar, 1979; Soames, 1982; Heim, 1983b; van der Sandt, 1992; Geurts, 1999; Beaver, 2001; Schlenker, 2008; Beaver & Clark, 2008) among many many others...

Presupposition

- **Presupposition** is part of the total meaning conveyed by an utterance.
- There is a huge literature on the matter: (Frege, 1892; Russell, 1905; Strawson, 1950; Ducrot, 1972; Stalnaker, 1974; Karttunen, 1974; Karttunen & Peters, 1979; Lewis, 1979; Gazdar, 1979; Soames, 1982; Heim, 1983b; van der Sandt, 1992; Geurts, 1999; Beaver, 2001; Schlenker, 2008; Beaver & Clark, 2008) among many many others...
- Core properties:
    - **Truth-value gap**: if the content of the presupposition is not true, it is difficult to judge whether the whole utterance is true or not.
    - Conventional **triggers**: definite descriptions, *it*-clefts, factive and change of state verbs...
    - **Projection** out of contexts that usually affect the truth-conditions:

(6) a. Ritchie **knows** that Lemmy plays the bass.
b. Ritchie does not **know** that Lemmy plays the bass.
c. Does Ritchie **know** that Lemmy plays the bass?
d. Maybe Ritchie **knows** that Lemmy plays the bass.
e. $\rightsquigarrow_{psp}$ Lemmy plays the bass.

The Projection Problem of Presupposition

- In some cases, the presupposition does not project.

(7) a. If France has king, the king of France is bald.
b. $\not\rightsquigarrow_{psp}$ France has a king.

(8) a. If Lemmy plays the bass, Ritchie knows it.
b. $\not\rightsquigarrow_{psp}$ Lemmy plays the bass.

The Projection Problem of Presupposition

- In some cases, the presupposition does not project.

(7) a. If France has king, the king of France is bald.
b. $\underset{psp}{\not\rightsquigarrow}$France has a king.

(8) a. If Lemmy plays the bass, Ritchie knows it.
b. $\underset{psp}{\not\rightsquigarrow}$Lemmy plays the bass.

- **Projection problem**: predicting the presuppositions of complex sentences from the presuppositions of their parts (Heim, 1983b).

The Projection Problem of Presupposition

- In some cases, the presupposition does not project.

(7) a. If France has king, the king of France is bald.
b. $\underset{psp}{\not\rightsquigarrow}$France has a king.

(8) a. If Lemmy plays the bass, Ritchie knows it.
b. $\underset{psp}{\not\rightsquigarrow}$Lemmy plays the bass.

- **Projection problem**: predicting the presuppositions of complex sentences from the presuppositions of their parts (Heim, 1983b).
- Presupposition projection is one of the two cornerstones (along with "donkey" anaphora) of the **dynamic turn** in semantics (Schlenker, 2008).

Dynamic Semantics

- **Dynamic semantics slogan**: meanings are **context change potentials** (rather than truth conditions) (Kamp, 1981; Heim, 1983a,b)

Dynamic Semantics

- **Dynamic semantics slogan**: meanings are **context change potentials** (rather than truth conditions) (Kamp, 1981; Heim, 1983a,b)
- Propositions are evaluated against **dynamic contexts**
- A recent formalization (Klinedinst & Rothschild, 2012; Lassiter, 2012a):
  - Propositions are evaluated against a context c, a world w and an information state s

    (9) $[\![\text{It's raining.}]\!]^{c,w,s}$ is true iff it's raining in w at the location in c

  - Information states are **sets of world** to which some phenomena are sensitive, e.g. it gives the domain quantification for epistemic modals:

    (10) $[\![\text{It might be raining}]\!]^{c,w,s}$ is true iff there is a world $w' \in s$ such that $[\![\text{It is raining}]\!]^{c,w',s} = 1$

The satisfaction theory of presupposition

- Presupposition projection is handled by a **usage constraint**:

(11) If φ is a (possibly) complex sentence with atomic parts $q_1, \ldots, q_n$ having semantic presuppositions $\underline{q_1}, \ldots, \underline{q_n}$ occurring in local information states $s_1, \ldots, s_n$, then φ should not be used unless $s_1 \subseteq \underline{q_1} \wedge \cdots \wedge s_n \subseteq \underline{q_n}$.

The satisfaction theory of presupposition

- Presupposition projection is handled by a **usage constraint**:

(11) If φ is a (possibly) complex sentence with atomic parts $q_1, \ldots, q_n$ having semantic presuppositions $\underline{q_1}, \ldots, \underline{q_n}$ occurring in local information states $s_1, \ldots, s_n$, then φ should not be used unless $s_1 \subseteq \underline{q_1} \wedge \cdots \wedge s_n \subseteq \underline{q_n}$.

- Truth-conditions for the logical conjunction **and**:

$[\![\varphi \wedge \psi]\!]^{c,s,w} = 1$ iff $[\![\varphi]\!]^{c,s,w} = 1$ and $[\![\psi]\!]^{c,\boxed{s+\varphi},w} = 1$

The satisfaction theory of presupposition

- Presupposition projection is handled by a **usage constraint**:

(11) If φ is a (possibly) complex sentence with atomic parts $q_1, \ldots, q_n$ having semantic presuppositions $\underline{q_1}, \ldots, \underline{q_n}$ occurring in local information states $s_1, \ldots, s_n$, then φ should not be used unless $s_1 \subseteq \underline{q_1} \wedge \cdots \wedge s_n \subseteq \underline{q_n}$.

- Truth-conditions for the logical conjunction **and**:

$[\![\varphi \wedge \psi]\!]^{c,s,w} = 1$ iff $[\![\varphi]\!]^{c,s,w} = 1$ and $[\![\psi]\!]^{c,\boxed{s+\varphi},w} = 1$

(12) Lemmy plays the bass and Ritchie knows it.

  - The second conjunct is evaluated in a context to which the first conjunct has been added, which validates the presupposition.

The satisfaction theory of presupposition

- Presupposition projection is handled by a **usage constraint**:

(11) If φ is a (possibly) complex sentence with atomic parts $q_1, \ldots, q_n$ having semantic presuppositions $\underline{q_1}, \ldots, \underline{q_n}$ occurring in local information states $s_1, \ldots, s_n$, then φ should not be used unless $s_1 \subseteq \underline{q_1} \wedge \cdots \wedge s_n \subseteq \underline{q_n}$.

- Truth-conditions for the logical conjunction **and**:

$$[\![\varphi \wedge \psi]\!]^{c,s,w} = 1 \text{ iff } [\![\varphi]\!]^{c,s,w} = 1 \text{ and } [\![\psi]\!]^{c,\boxed{s+\varphi},w} = 1$$

(12) Lemmy plays the bass and Ritchie knows it.

- The second conjunct is evaluated in a context to which the first conjunct has been added, which validates the presupposition.
- Without that information, either the presupposition has to be accommodated (Lewis, 1979) or the utterance is not acceptable (cf. usage constraint).

The case of conditionals

(13) $[\![\varphi \to \psi]\!]^{c,s,w} = 1$ iff $[\![\varphi]\!]^{c,s,w} = 0$ or $[\![\psi]\!]^{c,\,s+\varphi\,,w} = 1$

- A conditional sentence can only be used truthfully if the context and the antecedent entail the presupposition of the consequent.

The case of conditionals

(13) $[\![\varphi \rightarrow \psi]\!]^{c,s,w} = 1$ iff $[\![\varphi]\!]^{c,s,w} = 0$ or $[\![\psi]\!]^{c,\,s+\varphi\,,w} = 1$

- A conditional sentence can only be used truthfully if the context and the antecedent entail the presupposition of the consequent.
- This can be reformulated as saying that s must entail $\varphi \rightarrow \underline{\psi}$
- In other terms, a conditional *if* φ *then* ψ presupposes *if* φ *then* $\underline{\psi}$.

The case of conditionals

(13) $[\![\varphi \to \psi]\!]^{c,s,w} = 1$ iff $[\![\varphi]\!]^{c,s,w} = 0$ or $[\![\psi]\!]^{c,\boxed{s+\varphi},w} = 1$

- A conditional sentence can only be used truthfully if the context and the antecedent entail the presupposition of the consequent.
- This can be reformulated as saying that s must entail $\varphi \to \underline{\psi}$
- In other terms, a conditional *if* φ *then* ψ presupposes *if* φ *then* $\underline{\psi}$.
- This explains the apparent suspension of presuppositions:

(14) a. If Lemmy plays the bass then Ritchie knows it.
b. $\underset{psp}{\rightsquigarrow}$If Lemmy plays the bass then Lemmy plays the bass.

The proviso problem (Geurts, 1996)

- The previous prediction appears correct in some cases:

(15) a. If John is a diver, he will bring his wetsuit.
b. $\underset{psp}{\rightsquigarrow}$If John is a diver, he owns a wetsuit.

The proviso problem (Geurts, 1996)

- The previous prediction appears correct in some cases:

(15) a. If John is a diver, he will bring his wetsuit.
b. $\underset{psp}{\rightsquigarrow}$If John is a diver, he owns a wetsuit.

- However, in others, the predicted presupposition appears too weak, everything seems to project:

(16) a. If Lemmy forgot the concert, his manager will be angry.
b. $\underset{psp}{\not\rightsquigarrow}$If Lemmy forgot the concert, he has a manager.
c. $\underset{psp}{\rightsquigarrow}$Lemmy has a manager.

The proviso problem (Geurts, 1996)

- The previous prediction appears correct in some cases:

(15) a. If John is a diver, he will bring his wetsuit.
b. $\underset{psp}{\rightsquigarrow}$ If John is a diver, he owns a wetsuit.

- However, in others, the predicted presupposition appears too weak, everything seems to project:

(16) a. If Lemmy forgot the concert, his manager will be angry.
b. $\underset{psp}{\not\rightsquigarrow}$ If Lemmy forgot the concert, he has a manager.
c. $\underset{psp}{\rightsquigarrow}$ Lemmy has a manager.

- Usual accounts invoke a **strengthening mechanism** based on notions such as relevance (Singh, 2007)
  - This is usually ad-hoc and too powerful (Schlenker, 2011)

Probabilistic dynamic semantics

- Lassiter (2012a) proposes to use the same probabilistic semantics as for gradable epistemic modals:
- Information states (s) serve both as modal bases and as the information states against which presuppositions are tested.

Probabilistic dynamic semantics

- Lassiter (2012a) proposes to use the same probabilistic semantics as for gradable epistemic modals:
- Information states (s) serve both as modal bases and as the information states against which presuppositions are tested.
- Informations states are treated as **probability measures** on sets of worlds.
    - In other terms: speakers have a notion of the probability of the worlds they judge possible.
    - The probabilities are the same ones that the cognitive system manipulates for other tasks such as reasoning.

Probabilistic dynamic semantics

- Lassiter (2012a) proposes to use the same probabilistic semantics as for gradable epistemic modals:
- Information states (s) serve both as modal bases and as the information states against which presuppositions are tested.
- Informations states are treated as **probability measures** on sets of worlds.
  - In other terms: speakers have a notion of the probability of the worlds they judge possible.
  - The probabilities are the same ones that the cognitive system manipulates for other tasks such as reasoning.
- **Usage constraint** for atomic propositions

(17) A speaker should not utter p unless $P(\underline{p})$ meets or exceeds a high threshold θ according to her epistemic state, and she believes that her audience also assigns $\underline{p}$ at least probability θ.

Probabilistic conditionals

- New version of the semantics of conditionals:

(18) $[\![\varphi \to \psi]\!]^{c,P,w} = 1$ iff $[\![\varphi]\!]^{c,P,w} = 0$ or $[\![\psi]\!]^{c,\boxed{P(|\varphi)},w} = 1$

- Usage constraint for complex sentences:

(19) Appropriate use of φ requires that, for any atomic part p of φ with local information state P, (17) is satisfied.

Probabilistic conditionals

- New version of the semantics of conditionals:

(18) $[\![\varphi \to \psi]\!]^{c,P,w} = 1$ iff $[\![\varphi]\!]^{c,P,w} = 0$ or $[\![\psi]\!]^{c,\,P(|\varphi)\,,w} = 1$

- Usage constraint for complex sentences:

(19) Appropriate use of φ requires that, for any atomic part p of φ with local information state P, (17) is satisfied.

- For a conditional $p \to q$, the crucial condition is: $P(\underline{q}|p) > \theta$

Probabilistic conditionals

- New version of the semantics of conditionals:

(18) $[\![\varphi \rightarrow \psi]\!]^{c,P,w} = 1$ iff $[\![\varphi]\!]^{c,P,w} = 0$ or $[\![\psi]\!]^{c,\boxed{P(|\varphi)},w} = 1$

- Usage constraint for complex sentences:

(19) Appropriate use of φ requires that, for any atomic part p of φ with local information state P, (17) is satisfied.

- For a conditional $p \rightarrow q$, the crucial condition is: $P(\underline{q}|p) > \theta$
  - **Sidenote:** recall the discussion about Bayesian conditional reasoning, Oaksford & Chater (2007) argue that conditional probability is the probability of conditionals.
  - Therefore the above condition can be interpreted as a presupposition of the form **If** p **then** $\underline{q}$, i.e. the same as in the traditional approach.

Explaining the Proviso Problem

(20) If John is a diver, he will bring his wetsuit. =(15-a)

a. $P(\textit{John has a wetsuit}|\textit{John is a diver}) > \theta$

b. The two propositions are not independent,
i.e. $P(\underline{q}|p) \neq P(\underline{q})$

c. $\Rightarrow$ No "strengthening" of the presupposition

Explaining the Proviso Problem

(20) If John is a diver, he will bring his wetsuit. =(15-a)

a. $P(\textit{John has a wetsuit}|\textit{John is a diver}) > \theta$
b. The two propositions are not independent, i.e. $P(\underline{q}|p) \neq P(\underline{q})$
c. $\Rightarrow$ No "strengthening" of the presupposition

(21) If Lemmy forgot the concert, his manager will be angry. =(16-a)

a. $P(\textit{Lemmy has a manager}|\textit{Lemmy forgot the concert}) > \theta$
b. The two propositions are independent, i.e. $P(\underline{q}|p) = P(\underline{q})$
c. The usage condition is therefore equivalent to the simpler $P(\underline{q}) > \theta$ (= strengthened presupposition)

Taking stock: probabilistic semantics

- The probabilistic take on semantics is motivated on several grounds (gradable epistemic modals, intuitive adequacy to what beliefs are...)

Taking stock: probabilistic semantics

- The probabilistic take on semantics is motivated on several grounds (gradable epistemic modals, intuitive adequacy to what beliefs are...)
- It provides an **independent motivation** for the strengthening of some presuppositions in the proviso problem.

Taking stock: probabilistic semantics

- The probabilistic take on semantics is motivated on several grounds (gradable epistemic modals, intuitive adequacy to what beliefs are...)
- It provides an **independent motivation** for the strengthening of some presuppositions in the proviso problem.
- It can be formalized as a natural **extension** of **dynamic semantics**.

Taking stock: probabilistic semantics

- The probabilistic take on semantics is motivated on several grounds (gradable epistemic modals, intuitive adequacy to what beliefs are...)
- It provides an **independent motivation** for the strengthening of some presuppositions in the proviso problem.
- It can be formalized as a natural **extension** of **dynamic semantics**.
- **Caveat** of this approach:
  - The model proposed still relies on a partly truth-conditional approach to conditionals, although it is amended with probabilistic elements.
  - What predictions does it make for the Wason task? Is it compatible with the empirical data?
  - Technically, both could be combined, but the work remains to be done.

Other works

- Goodman & Lassiter (2014): propose a computational model for probabilistic with concrete programming examples based on a probabilistic version of the Lisp programming language.
- Jayez (2010); Colinet (2012): analyze **Free Choice Items** in terms of **entropy maximization** using probabilistic semantics.
- Various works using probabilistic semantics have been presented in a recent ESSLLI workshop (`http://www.bnlsp.ws/`)
  - Implicatures
  - Dialogue Act recognition
  - ...

Contents

Bayesian Natural Language Interpretation

- Zeevat (2014) proposes an account of natural language interpretation and production that has Bayesian characteristics.

Bayesian Natural Language Interpretation

- Zeevat (2014) proposes an account of natural language interpretation and production that has Bayesian characteristics.
- **Main objective**: deal with the **coordination problem**: "*how people in verbal communication manage to understand each other, i.e. how they reach* ***coordination***"

Bayesian Natural Language Interpretation

- Zeevat (2014) proposes an account of natural language interpretation and production that has Bayesian characteristics.
- **Main objective**: deal with the **coordination problem**: "*how people in verbal communication manage to understand each other, i.e. how they reach* ***coordination***"
- **Challenges**:
  - **Ambiguity**: how do agents converge quickly on a single reading?
  - The process appears **fast** and **effortless**.

Bayesian Natural Language Interpretation

- Zeevat (2014) proposes an account of natural language interpretation and production that has Bayesian characteristics.
- **Main objective**: deal with the **coordination problem**: "*how people in verbal communication manage to understand each other, i.e. how they reach* ***coordination***"
- **Challenges**:
  - **Ambiguity**: how do agents converge quickly on a single reading?
  - The process appears **fast** and **effortless**.
- The proposed account borrows from:
  - GPSG (Gazdar et al., 1985) and Optimality Theory for syntax.
  - Stochastic free categorial grammar and Bayes' rule for interpretation.
  - DRT (Kamp & Reyle, 1993) for semantic representation.

Characteristics of the proposed model

1. It predicts **coordination** on the speaker's meaning to be the standard occurrence in verbal communication, something that happens most of the time.
2. It predicts that both production and interpretation are **linear processes**.
3. Interpretation is filtered by **simulated production**.
4. Production is filtered by **simulated interpretation**.
5. It predicts a **gap** between production and interpretation (Clark & Hecht, 1983).
6. It explains **incremental interpretation** from the linearity of the interpretation process (Crain & Steedman, 1985).
7. It is good **syntax, semantics, pragmatics**, and good **AI**.

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:
  1. A system of **weighted cues**: morphemes, words, constructions, non-verbal signals... put forward a set of hypotheses about their meanings/interpretations.

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:
  1. A system of **weighted cues**: morphemes, words, constructions, non-verbal signals. . . put forward a set of hypotheses about their meanings/interpretations.
  2. **Prior** probabilities: probability of speaker's **communicative intentions** in a given context.

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:
  1. A system of **weighted cues**: morphemes, words, constructions, non-verbal signals... put forward a set of hypotheses about their meanings/interpretations.
  2. **Prior** probabilities: probability of speaker's **communicative intentions** in a given context.
  3. **Production** probabilities (=*likelihoods*): evaluated through **simulated production** from cued hypotheses.

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:
  1. A system of **weighted cues**: morphemes, words, constructions, non-verbal signals... put forward a set of hypotheses about their meanings/interpretations.
  2. **Prior** probabilities: probability of speaker's **communicative intentions** in a given context.
  3. **Production** probabilities (=*likelihoods*): evaluated through **simulated production** from cued hypotheses.
- Selection of the best hypothesis H_{opt} given a signal S is done via Bayes' rule:

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:
  1. A system of **weighted cues**: morphemes, words, constructions, non-verbal signals... put forward a set of hypotheses about their meanings/interpretations.
  2. **Prior** probabilities: probability of speaker's **communicative intentions** in a given context.
  3. **Production** probabilities (=*likelihoods*): evaluated through **simulated production** from cued hypotheses.
- Selection of the best hypothesis H_{opt} given a signal S is done via Bayes' rule:
  - $\forall i : P(H_i|S) \propto P(S|H_i) \times P(H_i)$

Bayesian Approach to Interpretation

- Zeevat rejects the standard assumption of parsing as the first step in natural language interpretation.
- He argues for a stochastic Bayesian interpretation system based on three components:
  1. A system of **weighted cues**: morphemes, words, constructions, non-verbal signals... put forward a set of hypotheses about their meanings/interpretations.
  2. **Prior** probabilities: probability of speaker's **communicative intentions** in a given context.
  3. **Production** probabilities (=*likelihoods*): evaluated through **simulated production** from cued hypotheses.
- Selection of the best hypothesis H_{opt} given a signal S is done via Bayes' rule:
  - $\forall i : P(H_i|S) \propto P(S|H_i) \times P(H_i)$
  - $H_{opt} = \underset{i}{argmax} P(S|H_i) \times P(H_i)$

Prior probabilities

- Prior probability depends on:
  1. The **conversational context** which assigns a probability to the conversational moves the speaker may be making given what happened before.
  2. The **model of the speaker**: how probable is it it that she would be giving this answer, express this question or wish.
  3. How probable is the content of the wish, information or question as a component of the world **given the context**.

Prior probabilities

- Prior probability depends on:
  1. The **conversational context** which assigns a probability to the conversational moves the speaker may be making given what happened before.
  2. The **model of the speaker**: how probable is it it that she would be giving this answer, express this question or wish.
  3. How probable is the content of the wish, information or question as a component of the world **given the context**.
- **Example**: John's wish to eat an apple is made less probable by:
  - being inappropriate at that particular point in the conversation
  - the fact that the audience would not hand John an apple
  - the absence of apples in the context

Application: marking rhetorical relations

(22) a. John fell. Bill pushed him. EXPLANATION
b. John fell **then** Bill pushed him. NARRATION

(23) a. John fell. Mary smiled at him. NARRATION
b. John fell **because** Mary smiled at him. EXPLANATION

- The same rhetorical relation can be expressed either without (22-a)-(23-a) or with an overt marker (22-b)-(23-b).

Application: marking rhetorical relations

(22)	a.	John fell. Bill pushed him.	EXPLANATION
	b.	John fell **then** Bill pushed him.	NARRATION
(23)	a.	John fell. Mary smiled at him.	NARRATION
	b.	John fell **because** Mary smiled at him.	EXPLANATION

- The same rhetorical relation can be expressed either without (22-a)-(23-a) or with an overt marker (22-b)-(23-b).
- **Problem**:
  1. Predict the **default** rhetorical relation
  2. Predict the **obligatoriness** of the marker when the intended relation is not the default

Application: marking rhetorical relations

(22) a. John fell. Bill pushed him. EXPLANATION
b. John fell **then** Bill pushed him. NARRATION

(23) a. John fell. Mary smiled at him. NARRATION
b. John fell **because** Mary smiled at him. EXPLANATION

- The same rhetorical relation can be expressed either without (22-a)-(23-a) or with an overt marker (22-b)-(23-b).
- **Problem**:
  1. Predict the **default** rhetorical relation
  2. Predict the **obligatoriness** of the marker when the intended relation is not the default
- **Note**: This is **not** a property of the relations.

Rhetorical relations (cont.)

- The choice of rhetorical relation is part of the global interpretation of an utterance.

Rhetorical relations (cont.)

- The choice of rhetorical relation is part of the global interpretation of an utterance.
- In the examples above:

Relation to John falling:	**Cause**	**Reaction**
Mary's smiling	Unlikely	Likely
Bill's pushing	Likely	Unlikely

Rhetorical relations (cont.)

- The choice of rhetorical relation is part of the global interpretation of an utterance.
- In the examples above:

Relation to John falling:	**Cause**	**Reaction**
Mary's smiling	Unlikely	Likely
Bill's pushing	Likely	Unlikely

- Bayesian interpretation explains which relation is preferred.

Rhetorical relations (cont.)

- The choice of rhetorical relation is part of the global interpretation of an utterance.
- In the examples above:

Relation to John falling:	**Cause**	**Reaction**
Mary's smiling	Unlikely	Likely
Bill's pushing	Likely	Unlikely

- Bayesian interpretation explains which relation is preferred.
- To explain the production of the marker, an hypothesis about **self-monitoring** is required (Levelt, 1983):
  - Speakers reason about the interpretation of their utterances
  - If they detect that the most probable interpretation differs from the intended one, they insert a marker.

Rhetorical relations (cont.)

- The choice of rhetorical relation is part of the global interpretation of an utterance.
- In the examples above:

Relation to John falling:	**Cause**	**Reaction**
Mary's smiling	Unlikely	Likely
Bill's pushing	Likely	Unlikely

- Bayesian interpretation explains which relation is preferred.
- To explain the production of the marker, an hypothesis about **self-monitoring** is required (Levelt, 1983):
  - Speakers reason about the interpretation of their utterances
  - If they detect that the most probable interpretation differs from the intended one, they insert a marker.
- The global picture is slightly more complex due to the existence of **soft-fringe** cases (Winterstein & Zeevat, 2012).

Contents

Contents

What is an argument?

Most treatments of argumentation (e.g. in philosophy, AI, psychology or linguistics) agree on the following:

- An argument is an attempt to **persuade** an agent . . .

What is an argument?

Most treatments of argumentation (e.g. in philosophy, AI, psychology or linguistics) agree on the following:

- An argument is an attempt to **persuade** an agent . . .
- An argument targets a **conclusion** (a **goal**)

What is an argument?

Most treatments of argumentation (e.g. in philosophy, AI, psychology or linguistics) agree on the following:

- An argument is an attempt to **persuade** an agent . . .
- An argument targets a **conclusion** (a **goal**)
- An argument is potentially **defeasible**, i.e. arguments can:

What is an argument?

Most treatments of argumentation (e.g. in philosophy, AI, psychology or linguistics) agree on the following:

- An argument is an attempt to **persuade** an agent . . .
- An argument targets a **conclusion** (a **goal**)
- An argument is potentially **defeasible**, i.e. arguments can:
  - be compared

What is an argument?

Most treatments of argumentation (e.g. in philosophy, AI, psychology or linguistics) agree on the following:

- An argument is an attempt to **persuade** an agent ...
- An argument targets a **conclusion** (a **goal**)
- An argument is potentially **defeasible**, i.e. arguments can:
  - be compared
  - undercut, refute, undermine each other

What is an argument?

Most treatments of argumentation (e.g. in philosophy, AI, psychology or linguistics) agree on the following:

- An argument is an attempt to **persuade** an agent . . .
- An argument targets a **conclusion** (a **goal**)
- An argument is potentially **defeasible**, i.e. arguments can:
  - be compared
  - undercut, refute, undermine each other
  - an argument has a given **strength** in favor of its conclusion

What is a good argument?

What is a good argument?

- **Classical view**: a good argument is **logically valid**
  - it is an acceptable form of deduction or induction
  - it avoids fallacies and non-valid reasoning

What is a good argument?

- **Classical view**: a good argument is **logically valid**
  - it is an acceptable form of deduction or induction
  - it avoids fallacies and non-valid reasoning
- **Practical view**: an argument is as good as it is **persuasive**.

What is a good argument?

- **Classical view**: a good argument is **logically valid**
  - it is an acceptable form of deduction or induction
  - it avoids fallacies and non-valid reasoning
- **Practical view**: an argument is as good as it is **persuasive**.
- In Bayesian terms: a good argument **raises the degree of belief** in its conclusion.
- This can be achieved in any way, as long as it is effective.
  - Hahn & Oaksford (2007): fallacies such as the argument from ignorance or the *petitio principii* can prove quite convincing in the right situation.

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):
  - A usual measure is **relevance** (not the same as in Relevance Theory (Sperber & Wilson, 1986; Merin, 1999)).

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):
  - A usual measure is **relevance** (not the same as in Relevance Theory (Sperber & Wilson, 1986; Merin, 1999)).
  - p is an argument for H iff $r(p, H) > 0$, the higher $r(p, H)$ the better the argument.

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):
  - A usual measure is **relevance** (not the same as in Relevance Theory (Sperber & Wilson, 1986; Merin, 1999)).
  - p is an argument for H iff $r(p, H) > 0$, the higher $r(p, H)$ the better the argument.
  - If $r(p, H)$ is negative, then p is a counter-argument for H.

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):
  - A usual measure is **relevance** (not the same as in Relevance Theory (Sperber & Wilson, 1986; Merin, 1999)).
  - p is an argument for H iff $r(p, H) > 0$, the higher $r(p, H)$ the better the argument.
  - If $r(p, H)$ is negative, then p is a counter-argument for H.
- The Bayesian treatment of argumentation might appear rather trivial:

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):
  - A usual measure is **relevance** (not the same as in Relevance Theory (Sperber & Wilson, 1986; Merin, 1999)).
  - p is an argument for H iff $r(p, H) > 0$, the higher $r(p, H)$ the better the argument.
  - If $r(p, H)$ is negative, then p is a counter-argument for H.
- The Bayesian treatment of argumentation might appear rather trivial:
  - Everything is handled by the **update mechanism**, captured via **conditionalization**, supposing that priors and joint probability distributions are known.

Argumentation in Semantics and Pragmatics

- In technical terms: an utterance of content p is an argument for a conclusion H iff $P(H|p) > P(H)$.
- The strength of an argument can be measured by a variety of means (Merin, 1999; van Rooij, 2004):
  - A usual measure is **relevance** (not the same as in Relevance Theory (Sperber & Wilson, 1986; Merin, 1999)).
  - p is an argument for H iff $r(p, H) > 0$, the higher $r(p, H)$ the better the argument.
  - If $r(p, H)$ is negative, then p is a counter-argument for H.
- The Bayesian treatment of argumentation might appear rather trivial:
  - Everything is handled by the **update mechanism**, captured via **conditionalization**, supposing that priors and joint probability distributions are known.
  - Therefore argumentation is just some **side effect** of the more general probabilistic take on meaning.

Linguistic Argumentation

- Anscombre & Ducrot (1983) fostered an argumentative approach to discourse:

> *The argumentative possibilities in a discourse are tied to the global linguistic structure of the utterances and not just to the content they convey.*

Linguistic Argumentation

- Anscombre & Ducrot (1983) fostered an argumentative approach to discourse:

  > *The argumentative possibilities in a discourse are tied to the global linguistic structure of the utterances and not just to the content they convey.*

- (24-a) and (24-b) have the same informational content, but (24-a) is a better argument for selling an insurance plan:

(24) a. Starting at only 29.9$ a month!
b. At least 29.9$ a month!

Linguistic Argumentation

- Anscombre & Ducrot (1983) fostered an argumentative approach to discourse:

  > *The argumentative possibilities in a discourse are tied to the global linguistic structure of the utterances and not just to the content they convey.*

- (24-a) and (24-b) have the same informational content, but (24-a) is a better argument for selling an insurance plan:

(24) a. Starting at only 29.9$ a month!
b. At least 29.9$ a month!

- **Hypothesis**: the semantic contribution of some linguistic items is best described in argumentative terms.
  - The description of those items can be done in probabilistic terms.

Contents

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is “disputed” by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is "disputed" by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is "disputed" by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

(25) a. Lemmy smokes but is in very good health.

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is "disputed" by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

(25) a. Lemmy smokes but is in very good health.
Lemmy is tall but Lars is short.

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is “disputed” by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

(25) a. Lemmy smokes but is in very good health.
Lemmy is tall but Lars is short.

b. • This **pivot** inference has different status:

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is "disputed" by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

(25) a. Lemmy smokes but is in very good health.
Lemmy is tall but Lars is short.

b. • This **pivot** inference has different status:
  - **Relevance theory**: an assumption made **accessible** by the first conjunct (Blakemore, 2002).

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is "disputed" by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

(25) a. Lemmy smokes but is in very good health.
Lemmy is tall but Lars is short.

b. • This **pivot** inference has different status:
  - **Relevance theory**: an assumption made **accessible** by the first conjunct (Blakemore, 2002).
  - **LDRT**: an inference of the same type as **particularized implicatures** (Spenader & Maier, 2009).

Adversative conjunctions: background

- The meaning of adversative connectives like ***but*** is often described in terms of **contrast** (Lakoff, 1971).
- **Inferential** approaches consider that the semantics of ***but*** always involve some kind of inference that is "disputed" by its conjuncts (Anscombre & Ducrot, 1977; Winterstein, 2012).

(25) a. Lemmy smokes but is in very good health.
Lemmy is tall but Lars is short.

b. • This **pivot** inference has different status:
  - **Relevance theory**: an assumption made **accessible** by the first conjunct (Blakemore, 2002).
  - **LDRT**: an inference of the same type as **particularized implicatures** (Spenader & Maier, 2009).
  - **Argumentation**: cf. infra.

Problematic examples

(26) a. Lemmy plays the bass, but he's the only one.
b. Lemmy plays the bass, but he's not the only one.

- In (26) ***but*** connects its first conjunct p with both q and $\neg q$.

Problematic examples

(26) a. Lemmy plays the bass, but he's the only one.
b. Lemmy plays the bass, but he's not the only one.

- In (26) ***but*** connects its first conjunct p with both q and $\neg q$.
- **Puzzle**: how can both q and $\neg q$ contrast with p?

Problematic examples

(26) a. Lemmy plays the bass, but he's the only one.
b. Lemmy plays the bass, but he's not the only one.

- In (26) ***but*** connects its first conjunct p with both q and $\neg q$.
- **Puzzle**: how can both q and $\neg q$ contrast with p?
  - This is not compatible with an analysis of the pivot as an implicature: implicatures based on a single utterance **cannot be contradictory**.

Problematic examples

(26) a. Lemmy plays the bass, but he's the only one.
b. Lemmy plays the bass, but he's not the only one.

- In (26) ***but*** connects its first conjunct p with both q and $\neg q$.
- **Puzzle**: how can both q and $\neg q$ contrast with p?
  - This is not compatible with an analysis of the pivot as an implicature: implicatures based on a single utterance **cannot be contradictory**.
  - **Relevance Theory**: the same utterance can make contradictory propositions accessible, however in (27) the **quantity implicature** of the first conjunct is accessible and should be able to serve as pivot:

Problematic examples

(26) a. Lemmy plays the bass, but he's the only one.
b. Lemmy plays the bass, but he's not the only one.

- In (26) ***but*** connects its first conjunct p with both q and $\neg q$.
- **Puzzle**: how can both q and $\neg q$ contrast with p?
  - This is not compatible with an analysis of the pivot as an implicature: implicatures based on a single utterance **cannot be contradictory**.
  - **Relevance Theory**: the same utterance can make contradictory propositions accessible, however in (27) the **quantity implicature** of the first conjunct is accessible and should be able to serve as pivot:

    (27) #Lemmy ate some of the cookies, but all of them.

Problematic examples

(26) a. Lemmy plays the bass, but he's the only one.
b. Lemmy plays the bass, but he's not the only one.

- In (26) ***but*** connects its first conjunct p with both q and $\neg q$.
- **Puzzle**: how can both q and $\neg q$ contrast with p?
  - This is not compatible with an analysis of the pivot as an implicature: implicatures based on a single utterance **cannot be contradictory**.
  - **Relevance Theory**: the same utterance can make contradictory propositions accessible, however in (27) the **quantity implicature** of the first conjunct is accessible and should be able to serve as pivot:

    (27) #Lemmy ate some of the cookies, but all of them.

  - **Note:** the pair in (26) is also problematic for non-inferential approaches to ***but***.

The argumentative meaning of but

- Anscombre & Ducrot (1977): an utterance "*p* ***but*** *q*" is such that:
  - p argues for a conclusion H
  - q argues against H, i.e. for $\neg H$
  - q must be a better argument for $\neg H$ than p is for H
- In probabilistic terms:
  - $r(p, H) > 0$
  - $r(q, H) < 0$
  - $|r(q, H)| > |r(p, H)|$

The argumentative meaning of but

- Anscombre & Ducrot (1977): an utterance "*p* ***but*** *q*" is such that:
  - p argues for a conclusion H
  - q argues against H, i.e. for $\neg H$
  - q must be a better argument for $\neg H$ than p is for H
- In probabilistic terms:
  - $r(p, H) > 0$
  - $r(q, H) < 0$
  - $|r(q, H)| > |r(p, H)|$
- **Example**:

(28) This car is nice but expensive.

  - $H =$ *We should buy the car*
  - p makes H more probable
  - q makes H less probable and "wins" over p: the speaker will (probably) not buy the car after uttering (28).

Potential goals

(29) Lemmy plays the bass.

- The set of **potential goals** of (29) is $\mathcal{G}_p = \{H | r(p, H) > 0\}$.

Potential goals

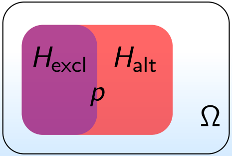
(29) Lemmy plays the bass.

- The set of **potential goals** of (29) is $\mathcal{G}_p = \{H | r(p, H) > 0\}$.
- Some elements of $\mathcal{G}_p$ are **context dependent**.

Potential goals

(29) Lemmy plays the bass.

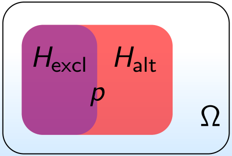
- The set of **potential goals** of (29) is $\mathcal{G}_p = \{H|r(p,H) > 0\}$.
- Some elements of $\mathcal{G}_p$ are **context dependent**.
- Others are **“mechanically” present**, notably:
  - H_{excl} = *Lemmy is the only one who plays the bass*
  - H_{alt} = *Lemmy is not the only one who plays the bass*



Potential goals

(29) Lemmy plays the bass.

- The set of **potential goals** of (29) is $\mathcal{G}_p = \{H | r(p, H) > 0\}$.
- Some elements of $\mathcal{G}_p$ are **context dependent**.
- Others are **"mechanically" present**, notably:
  - H_{excl} = *Lemmy is the only one who plays the bass*
  - H_{alt} = *Lemmy is not the only one who plays the bass*

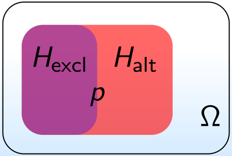


- H_{alt} and H_{excl} are the pivots in (26-a) and (26-b).

Potential goals

(29) Lemmy plays the bass.

- The set of **potential goals** of (29) is $\mathcal{G}_p = \{H | r(p, H) > 0\}$.
- Some elements of $\mathcal{G}_p$ are **context dependent**.
- Others are **"mechanically" present**, notably:
  - H_{excl} = *Lemmy is the only one who plays the bass*
  - H_{alt} = *Lemmy is not the only one who plays the bass*



- H_{alt} and H_{excl} are the pivots in (26-a) and (26-b).
- Even though they are contradictory, they both are potential goals for p.
- The probabilistic approach to argumentation has the right amount of **leeway** for what the goals can be.

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?
- The same reasoning as the one for H_{alt} and H_{excl} could be applied to the first conjunct of (30-a).

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?
- The same reasoning as the one for H_{alt} and H_{excl} could be applied to the first conjunct of (30-a).
- **some** and **all** form an argumentative scale (Ducrot, 1980):

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?
- The same reasoning as the one for H_{alt} and H_{excl} could be applied to the first conjunct of (30-a).
- **some** and **all** form an argumentative scale (Ducrot, 1980):
  - Replacing **some** by **all** in a sentence keeps the argumentative orientation

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?
- The same reasoning as the one for H_{alt} and H_{excl} could be applied to the first conjunct of (30-a).
- **some** and **all** form an argumentative scale (Ducrot, 1980):
  - Replacing **some** by **all** in a sentence keeps the argumentative orientation
  - If $r(\varphi_{[\text{some}]}, H) = R > 0$ then $r(\varphi_{[\text{all}]}, H) > R > 0$

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?
- The same reasoning as the one for H_{alt} and H_{excl} could be applied to the first conjunct of (30-a).
- **some** and **all** form an argumentative scale (Ducrot, 1980):
  - Replacing **some** by **all** in a sentence keeps the argumentative orientation
  - If $r(\varphi_{[\text{some}]}, H) = R > 0$ then $r(\varphi_{[\text{all}]}, H) > R > 0$
  - Negation reverses orientation, therefore $r(\varphi_{[\neg\text{all}]}, H) < 0$

The case of quantity implicatures

(30) a. #Lemmy solved some problems, but all of them.
b. Lemmy did not solve all the problems.

- What prevents (30-b) to be used as pivot for (30-a)?
- The same reasoning as the one for H_{alt} and H_{excl} could be applied to the first conjunct of (30-a).
- **some** and **all** form an argumentative scale (Ducrot, 1980):
    - Replacing **some** by **all** in a sentence keeps the argumentative orientation
    - If $r(\varphi_{[\text{some}]}, H) = R > 0$ then $r(\varphi_{[\text{all}]}, H) > R > 0$
    - Negation reverses orientation, therefore $r(\varphi_{[\neg\text{all}]}, H) < 0$
- **Hypothesis**: if H is such that $P(H|p) > 0$ but H and p are in argumentative opposition, then $H \notin \mathcal{G}_p$, i.e. p cannot be used as an argument for H.

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.
- **Bayesian** perspective:

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.
- **Bayesian** perspective:
  - We look for $H_{opt} = \underset{H_i \in \mathcal{G}_S}{argmax} P(S|H_i) \times P(H_i)$
  - Where S corresponds to the signal, including the argumentative constraints encoded by linguistic items such as ***but***.

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.
- **Bayesian** perspective:
  - We look for $H_{opt} = \underset{H_i \in \mathcal{G}_S}{argmax} P(S|H_i) \times P(H_i)$
  - Where S corresponds to the signal, including the argumentative constraints encoded by linguistic items such as ***but***.
- Where does $\mathcal{G}$, the set of goals, come from?

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.
- **Bayesian** perspective:
  - We look for $H_{opt} = \underset{H_i \in \mathcal{G}_S}{argmax} P(S|H_i) \times P(H_i)$
  - Where S corresponds to the signal, including the argumentative constraints encoded by linguistic items such as ***but***.
- Where does $\mathcal{G}$, the set of goals, come from?
  - For A&D this is not a question for linguistics but only a matter of world-knowledge.

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.
- **Bayesian** perspective:
  - We look for $H_{opt} = \underset{H_i \in \mathcal{G}_S}{argmax} P(S|H_i) \times P(H_i)$
  - Where S corresponds to the signal, including the argumentative constraints encoded by linguistic items such as ***but***.
- Where does $\mathcal{G}$, the set of goals, come from?
  - For A&D this is not a question for linguistics but only a matter of world-knowledge.
  - Formally, the set of goals whose probability is affected by an assertion is potentially **infinite**.

Abduction of the goal

- H_{alt} and H_{excl} satisfy the constraint imposed by ***but*** in (26-a) and (26-b).
- However, nothing has been said on the process of **abduction**, i.e. how speakers recognize that H_{alt} and H_{excl} are satisfactory.
- **Bayesian** perspective:
  - We look for $H_{opt} = \underset{H_i \in \mathcal{G}_S}{argmax} P(S|H_i) \times P(H_i)$
  - Where S corresponds to the signal, including the argumentative constraints encoded by linguistic items such as ***but***.
- Where does $\mathcal{G}$, the set of goals, come from?
  - For A&D this is not a question for linguistics but only a matter of world-knowledge.
  - Formally, the set of goals whose probability is affected by an assertion is potentially **infinite**.
  - **Hypothesis**: context, purely probabilistic effects, and discursive cues such as **information structure** define the contents of $\mathcal{G}$ (Winterstein, 2010, 2012).

Two levels of Bayesianism

- Argumentation uses two kinds of Bayesianism:
  1. **Probabilistic semantics**: utterances update degrees of belief.
  2. **Bayesian interpretation**: by reasoning on probabilistic change, the most likely argumentative goal is found. Linguistic cues constrain the space of possibilities for the argumentative goal.

Two levels of Bayesianism

- Argumentation uses two kinds of Bayesianism:
  1. **Probabilistic semantics**: utterances update degrees of belief.
  2. **Bayesian interpretation**: by reasoning on probabilistic change, the most likely argumentative goal is found. Linguistic cues constrain the space of possibilities for the argumentative goal.
- A basic tenet of argumentation is that two utterances with the same truth-conditional content can argue differently (cf. (24-a) vs. (24-b)).

Two levels of Bayesianism

- Argumentation uses two kinds of Bayesianism:
  1. **Probabilistic semantics**: utterances update degrees of belief.
  2. **Bayesian interpretation**: by reasoning on probabilistic change, the most likely argumentative goal is found. Linguistic cues constrain the space of possibilities for the argumentative goal.
- A basic tenet of argumentation is that two utterances with the same truth-conditional content can argue differently (cf. (24-a) vs. (24-b)).
- How to reconcile this with the update mechanism?

Same content, different arguments

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- Bayesian reasoning predicts that both versions of (31) should be **equally convincing**, since they both convey that:
  - 50% of the studies showed side-effects
  - 50% of the studies showed no side-effects

Same content, different arguments

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- Bayesian reasoning predicts that both versions of (31) should be **equally convincing**, since they both convey that:
  - 50% of the studies showed side-effects
  - 50% of the studies showed no side-effects
- A more refined approach to argumentative reasoning even predicts that both should argue for the dangerous nature of the drug (Hahn & Oaksford, 2007).

Same content, different arguments

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- Bayesian reasoning predicts that both versions of (31) should be **equally convincing**, since they both convey that:
  - 50% of the studies showed side-effects
  - 50% of the studies showed no side-effects
- A more refined approach to argumentative reasoning even predicts that both should argue for the dangerous nature of the drug (Hahn & Oaksford, 2007).
- Yet it seems that (31-b) is not a very good argument against using the drug.
  - This is confirmed by **experimental** results (Winterstein, 2014).

Levels of Meaning

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- In (31-a) and (31-b) the information is conveyed at different levels of meaning:

	(31-a)	(31-b)
50% showed side effects	at-issue	presupposition
50% did not show side effects	implicature	at-issue

Levels of Meaning

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- In (31-a) and (31-b) the information is conveyed at different levels of meaning:

	(31-a)	(31-b)
50% showed side effects	at-issue	presupposition
50% did not show side effects	implicature	at-issue

- **Hypothesis**: only the **at-issue** content is used for the belief update.

Levels of Meaning

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- In (31-a) and (31-b) the information is conveyed at different levels of meaning:

	(31-a)	(31-b)
50% showed side effects	at-issue	presupposition
50% did not show side effects	implicature	at-issue

- **Hypothesis**: only the **at-issue** content is used for the belief update.
  - Makes the right predictions for (31)

Levels of Meaning

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- In (31-a) and (31-b) the information is conveyed at different levels of meaning:

	(31-a)	(31-b)
50% showed side effects	at-issue	presupposition
50% did not show side effects	implicature	at-issue

- **Hypothesis**: only the **at-issue** content is used for the belief update.
  - Makes the right predictions for (31)
  - Confirmed by experiments on the meaning of **almost**

Levels of Meaning

(31) This drug is dangerous.

a. Half the studies showed it has side-effects.

b. Only half the studies showed it has side-effects.

- In (31-a) and (31-b) the information is conveyed at different levels of meaning:

	(31-a)	(31-b)
50% showed side effects	at-issue	presupposition
50% did not show side effects	implicature	at-issue

- **Hypothesis**: only the **at-issue** content is used for the belief update.
  - Makes the right predictions for (31)
  - Confirmed by experiments on the meaning of **almost**
  - Problematic status for **presupposition**, at least incompatible with assumptions of Lassiter (2012a).

General Summary

- Bayesian approaches are favored in a number of fields because of their **explanatory power**.
- Bayesian approaches in cognition make the (strong) assumption that **knowledge is probabilistic** in nature and that various modules of perception and reasoning access this knowledge to realize inferences of various sorts (vision, conditional reasoning. . .)
- There is no reason to assume that the **linguistic system** uses a different model for knowledge manipulation.
- Several works in semantics and pragmatics therefore propose accounts inspired by Bayesianism:
  - By using a **probabilistic** version of intensional logic
  - By using **Bayesian mechanisms** for interpretation
- Bayesian approaches remain a collection of unrelated works ; not all of them are **compatible** in the way they handle probability.

Thank You

References I

John R. Anderson, Ching-Fan Sheu (1995). "Causal inferences as perceptual judgments". In: *Memory & Cognition 23*, pp. 510–524.

Jean-Claude Anscombre, Oswald Ducrot (1977). "Deux mais en français". In: *Lingua 43*, pp. 23–40.

— (1983). *L'argumentation dans la langue*. Liège, Bruxelles: Pierre Mardaga.

David I. Beaver (2001). *Presupposition and Assertion in Dynamic Semantics*. CSLI Publications.

David I. Beaver, Brady Z. Clark (2008). *Sense and Sensitivity: How Focus determines meaning*. Wiley-Blackwell.

Diane Blakemore (2002). *Relevance and Linguistic Meaning. The semantics and pragmatics of discourse markers*. Cambridge: Cambridge University Press.

Rudolph Carnap (1950). *Logical Foundations of Probability*. Chicago: University of Chicago Press.

Eve V. Clark, Barbara F. Hecht (1983). "Comprehension, production, and language acquisition". In: *Annual Review of Psychology 34*, pp. 325–349.

Margot Colinet (2012). "Projective behavior and at-issueness of indefinite NPIs and FCIs". Communication at Going Romance 2012, Katholieke Universiteit Leuven, Belgium.

Richard Threlkeld Cox (1946). "Probability, frequency and reasonable expectation". In: *American journal of physics 14*, 1, pp. 1–13.

Steven Crain, Mark Steedman (1985). "On not being led up the garden path: The use of context by the psychological syntax processor". In: Lauri Karttunen, David Dowty, Arnold Zwicky (eds.), *Natural Language Parsing: Psychological, Computational and Theoretical Perspectives*, Cambridge: Cambridge University Press, pp. 320–358.

Oswald Ducrot (1972). *Dire et ne pas dire*. Paris: Hermann.

— (1980). *Les échelles argumentatives*. Les Éditions de Minuit.

References II

Gottlob FREGE (1892). "Über Sinn und Bedeutung". In: *Zeitschrift für Philosophie und philosophische Kritik 100*, pp. 25–50.

Gerald GAZDAR (1979). *Pragmatics: Implicature, Presupposition and Logical Form*. New York : Academic Press.

Gerald GAZDAR, Ewan KLEIN, Geoffrey PULLUM, Ivan SAG (1985). *Generalised Phrase Structure Grammar*. Basil Blackwell.

Bart GEURTS (1996). "Local satisfaction guaranteed". In: *Linguistics and Philosophy 19*, pp. 259–294.

— (1999). *Presuppositions and Pronouns*, vol. 3 of *Current Research in the Semantics/Pragmatics Interface*. Elsevier.

Gerd GIGERENZER (1991). "How to make cognitive illusions disappear: Beyond "heuristics and biases"". In: *European Review of Social Psychology 2*, 1, pp. 83–115.

Noah D. GOODMAN, Daniel LASSITER (2014). "Probabilistic Semantics and Pragmatics: Uncertainty in Language and Thought". In: Shalom LAPPIN, Chris FOX (eds.), *Handbook of Contemporary Semantics*, Oxford: Wiley-Blackwell. Draft version.

Ulrike HAHN, Mike OAKSFORD (2007). "The Rationality of Informal Argumentation: A Bayesian Approach to Reasoning Fallacies". In: *Psychological Review 114*, 3, pp. 704–732.

Irene HEIM (1983a). "File change semantics and the familiarity theory of definiteness". In: R. BÄUERLE, C. SCHWARZE, A. VON STECHOW (eds.), *Meaning, Use and Interpretation of Language*, Berlin: De Gruyter, pp. 164–189.

— (1983b). "On the projection problem for presuppositions". In: *Proceedings of WCCFL 2* pp. 114–125.

Jacques JAYEZ (2010). "Entropy and Free-choiceness". In: *Proceedings of the workshop on Alternative-Based Semantics*. Laboratoire de Linguistique de Nantes– Université de Nantes.

Philip .N. JOHNSON-LAIRD (1983). *Mental models*. Cambridge: Cambridge University Press.

Philip N. JOHNSON-LAIRD, Ruth M. J. BYRNE (2002). "Conditionals: A theory of meaning, pragmatics, and inference". In: *Psychological Review 109*, pp. 646–678.

References III

Hans Kamp (1981). "A Theory of Truth and Semantic Representation". In: Jeroen Groenendijk, Theo Janssen, martin Stokhof (eds.), *Formal Methods in the Study of Language*, Amsterdam: Mathematical Center Amsterdam, pp. 277–322.

Hans Kamp, Uwe Reyle (1993). *From Discourse to Logic*. Dordrecht: Kluwer.

Lauri Karttunen (1974). "Presuppositions and linguistic context". In: *Theoretical Linguistics 1*, pp. 181–194.

Lauri Karttunen, Stanley Peters (1979). "Conventional Implicatures in Montague Grammar". In: Choon-Kyu Oh, David Dineen (eds.), *Syntax and Semantics 11: Presupposition*, New York: Academic Press, pp. 1–56.

Christopher Kennedy, Louise McNally (2005). "Scale Structure, Degree Modification, and the Semantics of Gradable Predicates". In: *Language 81*, 2, pp. 345–381.

Nathan Klinedinst, Daniel Rothschild (2012). "Connectives without truth-tables". In: *Natural Language Semantics 20*, pp. 132–175.

Angelika Kratzer (1991). "Modality". In: Arnim von Stechow, Dieter Wunderlich (eds.), *Semantics: An International Handbook of Contemporary Research*, Berlin: de Gruyter, pp. 639–650.

Robin Lakoff (1971). "If's, And's and Buts about conjunction". In: Charles J. Fillmore, D. Terence Langendoen (eds.), *Studies in Linguistic Semantics*, New York: de Gruyter, pp. 114–149.

Pierre-Simon Laplace (1812). *Théorie analytique des probabilités*. Paris: Courcier.

Daniel Lassiter (2011a). "Gradable Epistemic Modals, Probability, and Scale Structure". In: Nan Li, David Lutz (eds.), *Semantics and Linguistic Theory (SALT) 20*. eLanguage, pp. 197–215.

— (2011b). *Measurement and Modality: The Scalar Basis of Modal Semantics*. Ph.D. thesis, NYU Linguistics.

— (2012a). "Presuppositions, provisos, and probability". In: *Semantics and Pragmatics 5*, 2, pp. 1–37.

— (2012b). "Probabilistic reasoning and statistical inference: An introduction (for linguists and philosophers)". Lecture notes, NASSLLI 2012 Bootcamp.

References IV

William J.M. Levelt (1983). "Monitoring and self-repair in speech". In: *Cognition 14*, pp. 41–104.

David Lewis (1979). "Scorekeeping in a language game". In: *Journal of Philosophical Logic 8*, pp. 339–359.

Craig R.M. McKenzie, Laurie A. Mikkelsen (2000). "The psychological side of Hempel's paradox of confirmation". In: *Psychonomic Bulletin & Review 7*, 2, pp. 360–366.

Arthur Merin (1999). "Information, Relevance and Social Decision-Making". In: L.S. Moss, J. Ginzburg, M. de Rijke (eds.), *Logic, Language, and computation*, Stanford: CSLI Publications, vol. 2, pp. 179–221.

Mike Oaksford, Nick Chater (2003). "Optimal data selection: Revision, review, and reevaluation". In: *Psychonomic Bulletin & Review 10*, 2, pp. 289–318.

— (2007). *Bayesian Rationality - the probabilistic approach to human reasoning*. Oxford: Oxford University Press.

— (2010). *Cognition and Conditionals: Probability and Logic in Human Thinking*. Oxford: Oxford University Press.

Judea Pearl (2009). *Causality: Models, Reasoning and Inference*. New York: Cambridge University Press, 2nd edn.

Karl R. Popper (1959). "The propensity interpretation of probability". In: *The British journal for the philosophy of science 10*, 37, pp. 25–42.

Frank P. Ramsey (1926). "Truth and Probability". In: R.B. Braithwaite (ed.), *The Foundations of Mathematics and other Logical Essays*, London: Kegan, Paul, Trench, Trubner & Co., chap. VII, pp. 156–198.

Lance J. Rips (1994). *The psychology of proof*. Cambridge, MA: MIT Press.

Robert van Rooij (2004). "Cooperative versus argumentative communication". In: *Philosophia Scientia 2*, pp. 195–209.

Bertrand Russell (1905). "On denoting". In: *Mind* .

Rob A. van der Sandt (1992). "Presupposition Projection as Anaphora Resolution". In: *Journal of Semantics 9*, pp. 333–377.

References V

Philippe Schlenker (2008). "Be Articulate: A Pragmatic Theory of Presupposition Projection". In: *Theoretical Linguistics 34*, pp. 157–212.

— (2011). "The proviso problem: A note". In: *Natural Language Semantics 19*, 4, pp. 395–422.

Lauren Schmidt (2009). *Meaning and compositionality as statistical induction of categories and constraints*. Ph.D. thesis, Massachusetts Institute of Technology.

Raj Singh (2007). "Formal alternatives as a solution to the proviso problem". In: *Proceedings of Semantics and Linguistic Theory (SALT) 17*. pp. 264–281.

Scott Soames (1982). "How Presuppositions are inherited : A Solution to the Projection Problem". In: *Linguistic Inquiry 13*, pp. 483–545.

Jennifer Spenader, Emar Maier (2009). "Contrast as denial in multi-dimensional semantics". In: *Journal of Pragmatics 41*, pp. 1707–1726.

Dan Sperber, Deirdre Wilson (1986). *Relevance: Communication and Cognition*. Oxford: Blackwell, 2nd edn.

Robert C. Stalnaker (1974). "Pragmatic Presuppositions". In: *Semantics and Philosophy* .

Peter F. Strawson (1950). "On referring". In: *Mind 59*, 235, pp. 320–344.

Joshua B. Tenenbaum, Charles Kemp, Thomas L. Griffiths, Noah D. Goodman (2011). "How to grow a mind: statistics, structure, and abstraction". In: *Science 331*, 6022, pp. 1279–1285.

Peter Cathcart Wason (1966). "Reasoning". In: B.M. Foss (ed.), *New horizons in psychology*, Harmondsworth: Penguin.

Grégoire Winterstein (2010). *La dimension probabiliste des marqueurs de discours. Nouvelles perspectives sur l'argumentation dans la langue*. Ph.D. thesis, Université Paris Diderot.

— (2012). "What **but**-sentences argue for: a modern argumentative analysis of **but**". In: *Lingua 122*, 15, pp. 1864–1885.

References VI

— (2014). “Layered Meanings and Bayesian Argumentation. The case of exclusives.” Accepté pour publication.

Grégoire Winterstein, Henk Zeevat (2012). “Empirical Constraints on Accounts of *too*”. In: *Lingua 122*, 15, pp. 1787–1800.

Fei Xu, Vashti Garcia (2008). “Intuitive statistics by 8-month-old infants”. In: *Proceedings of the National Academy of Sciences of the United States of America 105*, 13, pp. 5012–5015.

Seth Yalcin (2007). “Epistemic modals”. In: *Mind 116*, 464, pp. 983–1026.

Henk Zeevat (2014). *Language Production and Interpretation: Linguistics meets Cognition*. Leiden: Brill.